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Scalar Expectancy Theory and Weber's Law in Animal Timing

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A theory of temporal control is described which treats responding of animal subjects at asymptote under a variety of learning procedures. Subjects are viewed as making estimates of the time to reinforcement delivery using a scalar-timing process, which rescales estimates for different values of the interval being timed. Scalar-timing implies a constant coefficient of variation. Expectancies of reward based on these estimates are formed, and a discrimination between response alternatives is made by taking a ratio of their expectancies. In periodic schedules of reinforcement the discrimination is between local and overall expectancy of reward. In psychophysical studies of duration discrimination, the expectancy ratio reduces the likelihood ratio, and in conjunction with the scalar property, results in a general form of Weber's law. The psychometric choice function describing preference for different amounts and delays of reinforcement also results in a form of Weber's law.

Time is a ubiquitous variable in experiments on learning in animals. The very meaning of conditioning implies that subjects are sensitive to the forward pairing in time between conditional and unconditional stimuli, or responses and unconditional stimuli. Temporal arrangements of relevant stimulus events, of course, are required for any experimental preparation, and even when temporal regularity is degraded to render control by periodicity small, subjects nevertheless respond to average rates of stimulation in an orderly way (Herrnstein, 1970). When periodicity in the presentation of reinforcing events is the rule rather than the exception, time becomes a controlling variable and subjects evince temporal discriminations of the relevant events. The present article proposes a theoretical account of the way in which such temporal control modulates the performance of subjects at asymptote in a variety of learning situations. The theory stems in part from earlier work on timing in avoidance procedures (Gibbon, 1971, 1972) and depends centrally upon a simple relativistic property of time estimation. This property I have called *scalar-timing* in avoidance work, but it has been extant in different contexts and named differently by different investigators in animal learning.

Examples of this property are shown in Figure 1. The left panel shows response rate of pigeons on a fixed-interval (FI) schedule in which reinforcement is available only after a fixed period of time (T). Rates of responding are shown as a proportion of the maximum rate attained at the end of the fixed interval, at proportional points within the interval (t/T). The different curves were obtained from different fixed-interval durations spanning a very broad range ($\frac{1}{2}$ min to 50 min). The rising form of these functions exemplifies the "FI scallop." Responding is low at the start of the interval and increases as the time for reinforcement delivery is approached. The data are taken from Dews (1970), who describes them as reflecting "proportional timing," since rate at a given proportion of the interval is a constant proportion of the maximum rate attained at the

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end of the interval. The superposition of these functions means that this relation is independent of the absolute size of the interval.

The middle panel shows behavior of a rat under two fixed-interval shock schedules (LaBarbera & Church, 1974). Responding was supported on a variable-interval (VI) schedule of positive reinforcement, and response-independent shocks were presented at fixed intervals of time. The measure of responding is response rate at successive points within the FI. The two curves represent responding when the intershock interval was 1 min and when this interval was 2 min. Time within the FI is again scaled proportionally. The reduction in responding is approximately constant at a given proportion of the fixed interval.

The right panel represents avoidance latency distributions obtained in my laboratory from a rat working on a free-operant avoidance schedule at several different values of the response-shock interval (T). The data are the proportion of avoidances which exceed a given elapsed time (t) as a function of the proportion of the preshock interval (t/T). The four curves represent the data from four different values of the response-shock interval and have been adjusted by subtracting a small constant ($M = .87$) which may represent the motor time required to execute the response (Gibbon, 1971). Again, it appears that the time to reinforcement delivery is assessed in a proportional manner so that the same proportion of behavior occurs at a given proportion of the preshock interval, independently of the absolute size of that interval. Church and his colleagues (Libby & Church, 1974) have reported similar scalar timing in shuttle-box avoidance.

The scalar-timing hypothesis (Gibbon, 1971, 1972) proposed that behavior in these situations is directly related to estimates of the time to reinforcement. The estimates, in turn, were scale transforms of a "unit timer" appropriate to the estimation of one unit of time. Subjects

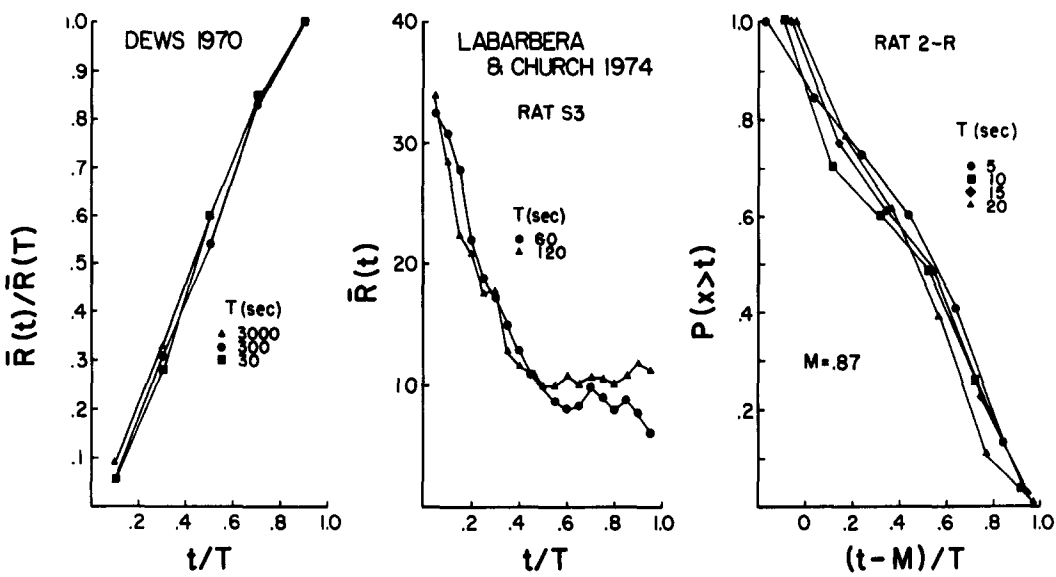

Figure 1. Three examples of relativistic timing behavior. The left panel (Dews, 1970) represents the response rate of pigeons under fixed-interval schedules of food reinforcement as a proportion of the terminal rate at the end of the fixed interval (FI). Relative rate ($\bar{R}$) is plotted at successive proportional points within the interval (t/T). Each function represents an FI value (T). The next panel (LaBarbera & Church, 1974) represents response rate of a rat on a variable-interval schedule of positive reinforcement during the period between successive shocks scheduled T sec apart. The circles are for $T = 60$ sec and diamonds are for $T = 120$ sec. The abscissa is again scaled in proportions of T. The right-hand panel represents avoidance latencies of a rat on a free-operant avoidance schedule. Data points are the proportion of avoidance latencies, x, exceeding t, when the response-shock interval (T), was either 5, 10, 15, or 20 sec. The curves have been shifted to the left by a small constant ($M = .87$), representing a fixed component in the avoidance latency unrelated to T.

differed in the efficiency with which they could estimate time, but were alike in that changes in the value of the time being estimated resulted in a simple scale transform of the unit timer. The theory did not specify how time estimates were translated into behavior, but only that the translation preserved the scalar property. A similar ambiguity is evident in "proportional timing," since the way in which timing is converted to response rates is not specified. In the theory to follow, a specific translation mechanism is proposed which regards behavior in time-correlated experimental paradigms as reflecting expectancies of reward based upon scalar time estimates of when reinforcement is due. Responding in simple reinforcement schedules is viewed as resulting from a discrimination between the current or local expectancy of reinforcement and the overall expectancy of reinforcement. Thus the theory is properly described as a discrimination theory of temporal control. Application of these ideas will be restricted to data from appetitive reinforcement procedures, since most of the work on choice and timing has used positive reinforcement and since much of the work on avoidance procedures has been already described (Gibbon, 1971, 1972).

The core of the scalar-timing hypothesis is that variance of time estimates increases with the square of the mean, and accordingly, the account focuses on the first two moments of distributional phenomena. The theory differs from previous quantitative accounts of choice in that a serious attempt is made to characterize variability in temporal control. It will be seen that different constructions of the timing process may be differentiated only when standard deviations as well as means are considered. Simple schedules of strictly periodic reinforcement are considered first and then variants of periodic schedules, in which reinforcement delivery depends to some extent on the subjects' behavior, are treated. Next, discrimination of duration is studied. The scalar hypothesis in conjunction with the expectancy proposal is shown to imply Weber's law in psychophysical procedures. Finally, choice between different delays and amounts of reinforcement is treated in a variety of experimental contexts.

Point Estimation

Fixed-Interval Reinforcement

In fixed-interval schedules of reinforcement, it is unlikely that responding directly reflects estimates of the actual time to reinforcement delivery, since responding typically begins considerably earlier in the interval than is necessary to pick up the single reinforcement delivered at its end. After sufficient training, subjects respond at a very low rate in the early portion of the interval and change abruptly to a high rate at about two thirds of the way into the interval and continue responding at this rate until reinforcement delivery (Schneider, 1969). The theory proposes that subjects make an estimate of when reinforcement is due and base an expectancy of reward on that value. As time in the interval elapses, expectancy increases. When momentary or "local" expectancy represents a discriminable improvement over overall expectancy of reinforcement, the high terminal rate of responding begins.

Expectancy. Expectancy of reinforcement is inversely related to the estimated time to reinforcement and may be thought of as an instantaneous rate-of-reinforcement estimate. The discrimination of improvement in expectancy is effected by taking a ratio of local or "immediate" expectancy to overall or "undiscriminated" expectancy. The expectancy ideas are depicted in Figure 2. In the top panel, a distribution of estimates of the fixed-interval time (T) is shown with a sample (x) indicated. In the bottom panel, on the same time axis, the expectancy formulation is described. Expectancy of reinforcement is defined as the total amount of expectancy that a single reinforcer will support, spread back uniformly over the estimated time to reinforcement. The total amount of expectancy or "hope" is scaled in units of amount of or access to the reinforcer, and is designated H. H is a motivational parameter which is assumed to vary directly with traditional incentive or drive operations. It may be thought of as a subjective unit of "value" or excitatory strength.

At the outset of the interval ($t=0$), expectancy is simply H/x. This value is indicated by the diagonal hatching and symbolized as $h(0)$. If, as depicted here, the mean of the estimation distribution is centered on T, $h(0)$

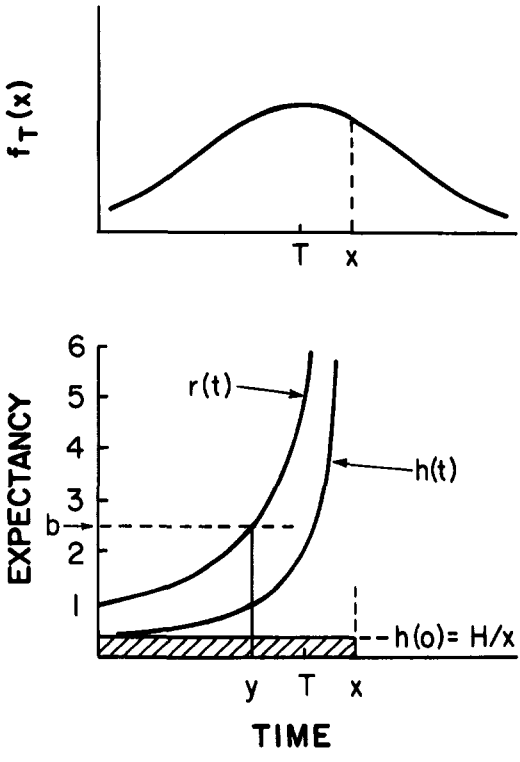

Figure 2. Estimates of fixed-interval time, T, (top panel) with a sample, x, indicated. Expectancies of reinforcement based upon that value are shown in the lower panel. The expectancy function $h(t)$, and the expectancy ratio $r(t)$, are hyperbolic in t and the point at which the expectancy ratio crosses the threshold, b, is indicated as y.

for the mean estimate is simply the reinforcement rate multiplied by the total expectancy or incentive value of the reinforcer, H. Expectancy at the beginning of each interval is, in this sense, an estimate of overall expectancy for a given amount and quality of reinforcement delivered at a rate of $1/T$.

As time elaspses within the interval, expectancy is continuously "updated" for the elapsed time, $t > 0$. It therefore increases in a hyperbolic manner since H is now spread over the remaining time between t and the estimate of reinforcement delivery, x. That is

$$h(t) = \frac{H}{x - t}. \qquad (1)$$

This is the lower function in Figure 2. Essentially this idea of a hyperbolic form for expectancy has been proposed on empirical grounds by Rachlin (1970), Rachlin and Green (1972), and Ainslie (1975; see also Navarick & Fantino, 1976) to account for "impulsiveness" and "self-control" at differing temporal proximities to reinforcement. A hyperbolic index of a similar kind has also been proposed by Staddon (1972b) as an organizing principle in schedules of reinforcement, following Jenkins' (1970) treatment of "relative proximity" (see also Shull & Brownstein, 1975).

The concept of expectancy is, of course, an old one in learning (e.g., Tinklepaugh, 1928) and has been applied with force to qualitative accounts of many features of animal learning (Bolles, 1970). The present formulation adds to the traditional concept of expectancy in the quantitative rationale with a motivational parameter, in the expectancy ratio comparator described below, and in the stochastic estimation process. The expectancy function defined by Equation 1 grows infinite as the expected time to reinforcement is approached, and thus the function is not defined for values of t larger than the estimate, x. This property means it is not possible to obtain an average expectancy function by integrating over x values. This difficulty is resolved, however, for the measures considered here, which depend upon expectancy attaining a threshold level.

Expectancy ratio. Expectancy of reinforcement increases inversely with the time remaining in the interreinforcement interval. Responding, however, emerges only when a discriminable or "worthwhile" improvement in expectancy is achieved. Responding in this account represents an option between timing locally within the interval, and not timing. As time in the interval elapses, subjects compare the local expectancy value, $h(t)$, with the estimate of the overall or undiscriminated expectancy of reward, $h(0)$. The comparison is made by taking the ratio of local to overall expectancy, $h(t)/h(0)$. Responding begins when this ratio exceeds a threshold value. That is, responding begins when

$$r(t) \equiv \frac{h(t)}{h(0)} = \frac{1}{1 - t/x} > b. \qquad (2)$$

Notice that the motivational parameter, H, drops out in the expectancy ratio, $r(t)$, but the hyperbolic character of the function is preserved. In Figure 2, the expectancy ratio cor-

responding to an estimate of x is shown and the point at which it meets the threshold, b, is indicated as y.

The assumption of a ratio comparator might be viewed as unnecessarily complicated to handle simple fixed-interval schedules. It substantively regards simple timing as a discrimination between "local" timing and what might be called "global" timing, or "rate" timing. Subjects ask, at any point within the interval, whether their expectancy of reward represents a sufficiently large increment over their expectancy of reward at the beginning of the interval to warrant responding.

In Figure 2, it is clear that when estimates of T are long, behavior is initiated late. From Equation 2, the time, y, at which responding emerges is proportional to the estimates,

$$y = (1 - 1/b)x. \quad (3)$$

The time value, y, at which responding changes from a low rate to a high rate (the "break point") is simply a rescaled estimate of when reinforcement is due. The expectancy-ratio rule means that the break point should be independent of motivational variables since H is eliminated in Equation 2. Evidence on this point will be discussed later.

Timing. When the interval being estimated is increased, estimates increase also. However, the manner in which that increase is effected varies in alternative constructions of the timing process. In Figure 3, the scalar-timing hypothesis is contrasted with four other possibilities. In the top panel, a hypothetical distribution of estimates of one unit of time is shown. The lower four panels illustrate the manner in which time estimates might change when the value being estimated is doubled. In the panel labeled "no timing," the distribution is identical to that for the unit timer and thus estimates are independent of the size of the interval. Such a process reflects elicitation rather than anticipation, since estimates are tied to the onset of the timing interval rather than its expiration. Elicited components in timing behavior are ubiquitous in free-operant avoidance procedures. Postshock "bursts" of responding (e.g., Boren, 1961) are common, and Neffinger and Gibbon (1975) have shown that elicited behavior under control of noncontingent shock interacts with contingency-

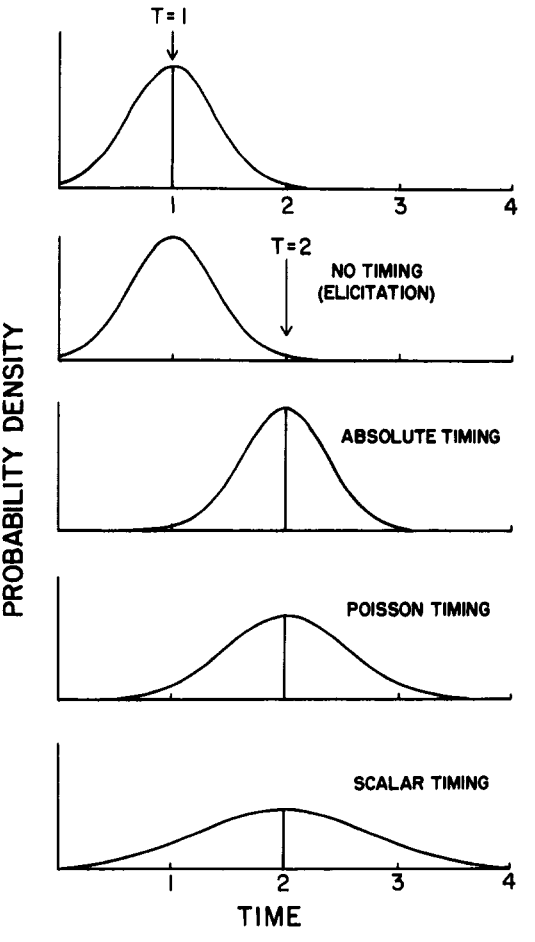

Figure 3. Time estimation distributions for one unit of time ($T = 1$, top panel) and for four alternative timing processes when the estimate is of two units ($T = 2$). In the no-timing case, there is no change in the form of the distribution. In the absolute-timing case the distribution moves laterally (linearly) with the value being timed. In the Poisson-timing case, estimates increase in a manner such that the mean and variance are proportional to T. The scalar-timing case (bottom panel) is a scale transform of the unit timer, so that the mean and standard deviation are proportional to T.

sensitive avoidance responding. In classical conditioning the alpha response probably has this elicited character (e.g., Boneau, 1958), and activity under periodic appetitive reinforcement (Killeen, 1975) may also reflect a reinforcement-elicited component. The no-timing case may thus be involved in much of the data analyzed here. However, for relatively large time intervals, elicitation represents a small contribution to the mean and variance of the

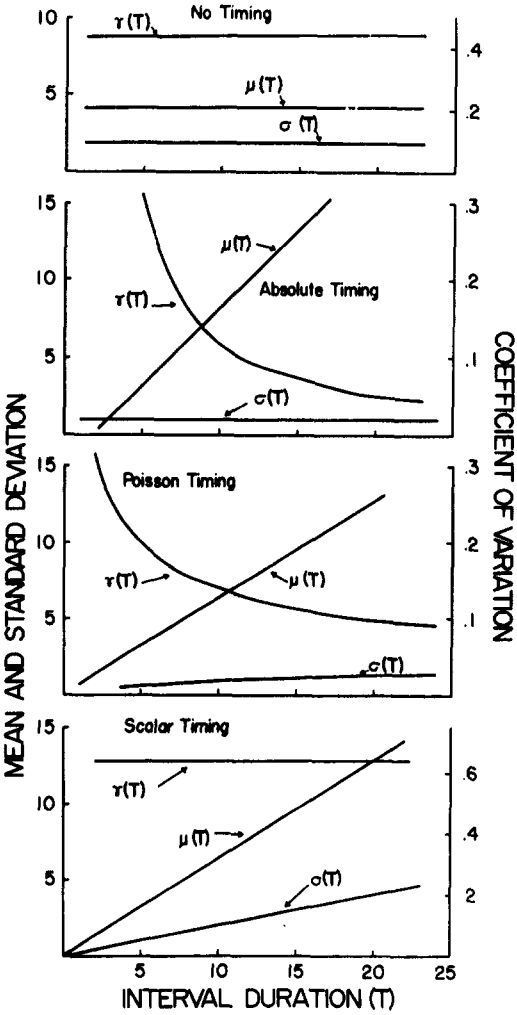

Figure 4. Mean, $\mu(T)$, standard deviation, $\sigma(T)$, and coefficient of variation, $\gamma(T)$, for the four alternatives. Each panel represents one of the four alternatives and mean and standard deviation functions are referred to the left ordinate, and the coefficient of variation is referred to the right ordinate. See text for further detail.

estimation distributions, and the account will focus on anticipatory behavior related to the end rather than to the beginning of the timing interval.

In the next panel, labeled "absolute timing," the timing distribution has been moved laterally with the value being timed so that variability around this value remains constant and a given deviation in absolute value from T has equal likelihood in the $T = 1$ and $T = 2$ cases. Such a process is linearly related to the interval being estimated. If one thinks of different time values as corresponding to different sensory input in a psychophysical context, absolute-timing represents Thurstone's Case V, in which the discriminal dispersions are constant (Thurstone, 1927a). Kristofferson (1976, see also Allan & Kristofferson, 1974) has shown that human timing for T below about .5 sec has just this character, while for larger intervals, variance increases with the time being estimated. Animal timing data, however, are generally not available for T values less than about 1 or 2 sec.

The next panel, labeled "Poisson-timing," represents the manner in which time estimates might shift if they depended upon a count of impulses generated by a Poisson source with a constant mean rate. Such a process is familiar as "neural counting" or "timing" (Getty, 1975; Luce & Green, 1972; McGill, 1967). When the interval being estimated is doubled, the new estimates are produced by simply doubling the number of impulses required. For Poisson-timing, estimation is more efficient at long than at short times, since the standard deviation increases as the square root of the mean ("Square Root Law").[1]

In the bottom panel, the scalar-timing process is shown (Gibbon, 1971, 1972). The distribution for estimates of two units of time is a simple scale transform of the unit timer distribution. In this process, the proportion of estimates below a given proportion of the timing interval remains constant. Letting $u = x/T$ for the unit timer,

$$f_T(x) = \frac{1}{T} f(u), \text{ and} \tag{4a}$$

$$F_T(x) = F(u), \tag{4b}$$

where $f_T(x)$, $f(u)$, and $F_T(x)$, $F(u)$ are the densities and distribution functions for estimates of T and unity, respectively (Gibbon, 1971).

These four alternatives differ in the way in which moments of the estimation distributions change with changes in the size of the timing interval. In Figure 4, predictions for the mean,

[1] In this example, a large sample Gaussian approximation to the Gamma distribution has been used, with the appropriate relations for mean and variance.

μ, standard deviation, σ, and coefficient of variation, $\gamma = \sigma/\mu$, are shown for each alternative. In the no-timing or elicitation process, there is no change in the mean or standard deviation (and hence, in the coefficient of variation) with changes in T. Absolute-timing means that the mean value increases linearly with T since estimates are simply $x = u + T - 1$, where u is the unit estimator. The function for the mean therefore has a slope of 1 and an intercept equal to the mean for the unit timer minus 1. The variance does not change and so the coefficient of variation, γ, varies inversely with T.

The Poisson process generates mean estimates which are proportional to T, since the impulse rate remains constant, but the count required for an estimate increases proportionally with T. The standard deviation, however, increases proportionally with the square root of T, so the coefficient of variation decreases inversely with $\sqrt{T}$.

Scalar-timing is shown in the bottom panel. The mean and standard deviation are both proportional to the interval being timed so that the coefficient of variation is constant.

$$\mu(T) = \mu T, \tag{5a}$$

$$\sigma(T) = \sigma T, \text{ and} \tag{5b}$$

$$\gamma(T) = \gamma, \tag{5c}$$

where, μ, σ, and γ are the statistics for the unit timer.

The four alternatives thus make very different predictions with respect to changes in the moments of the distributions with changes in T. Poisson-timing and scalar-timing are more realistic alternatives for relatively large T values, and they do not differ in their predictions for the mean value function. However, they do differ critically in predictions for the standard deviation and the coefficient of variation. The square root relation for the coefficient of variation in Poisson-timing means that "sensitivity" or efficiency of the estimation process increases with T. For the scalar-timing process, in contrast, γ remains constant and represents the fundamental sensitivity parameter of the timing process, comparable to the Weber fraction (cf. Treisman, 1963). This parameter indexes the efficiency of temporal resolution capacities and thus plays a role like d' for the equal variance case in signal detection theory (Green & Swets, 1966).

In the context of the expectancy account proposed above, the distribution of break points, y, (Equation 3) is a simple proportional transform of the distribution of estimates, x. Thus, for scalar-timing, the proportional relations for the mean and standard deviation (Equations 5a and 5b) hold, but with the constant of proportionality multiplied by $1 - 1/b$. The coefficient of variation (Equation 5c) remains unchanged. For the absolute-timing and Poisson-timing alternatives, the proportional translation for break points also simply changes the constants in the mean and standard deviation functions shown in Figure 4, and so a clear discrimination among these alternatives is possible since the form of the relationships are not changed.

Schneider (1969) studied fixed-interval responding in pigeons at a substantial range of values. Schneider and Neuringer (1972) studied a variant of the fixed-interval procedure in which the intervals were broken up into short periods of key-light on and key-light off, so that probability rather than rate of response was the dependent variable. In both cases, responding was consistently low early in the interval and changed abruptly to a high rate (Schneider, 1969) or a high probability (Schneider & Neuringer, 1972) later in the interval. Complete distributions of break points were published for one subject in each study, as well as the mean break point averaged over subjects at each FI value (T). These data are replotted on double log coordinates in Figure 5.[2] The functions for the mean are represented by circles, the standard deviation by triangles, and the coefficient of variation by squares. The two individual birds are on the left and the average for both studies of the mean break-point function is shown on the

[2] Throughout this article, unless otherwise stated, data not presented in tables were estimated from figures by the following procedure: Figures were photographed and enlarged to 8 × 10 inches (20.3 × 25.4 cm). The values for data points were then estimated using a Gerber scientific graphanalogue rule (Model GA-103) with variable spacing (a "scalar ruler"). The accuracy of these estimates appeared to be within 1% of the data values.

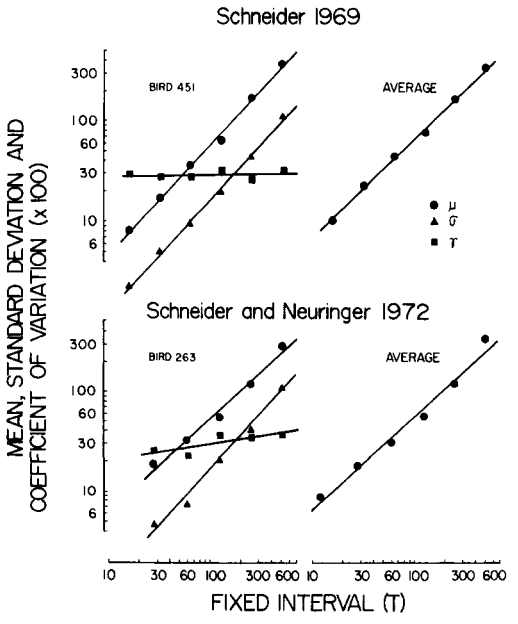

Figure 5. Mean (circles), standard deviation (triangles) and coefficient of variation (squares) for fixed-interval breakpoint distributions. Two individual birds from each study are shown on the left and the average mean breakpoint on the right. Both scales are logarithmic.

right ($N = 6$ for the Schneider, and $N = 5$ for the Schneider & Neuringer, studies).

The no-timing and absolute-timing hypotheses are clearly untenable in the light of the mean functions on the right, which show clear evidence of proportionality. The no-timing process would have a slope of 0 on this plot, and absolute-timing would have a concave down curvature for small T and a slope of 1 with $\log y = \log x$ for large T. Proportionality for the average break points translates here into linear functions with a slope of 1.0. The straight lines were obtained by least squares and the slopes are .996 and .940 for the Schneider and Schneider, and Neuringer data, respectively. The proportions of variance accounted for by these functions are .998 and .980, respectively. Slopes of 1.0 are approximated for the mean and standard deviation data of the two individuals on the left as well. Slopes for the means are 1.09 and .89, and for the sigmas are 1.11 and 1.19, for birds 451 and 263, respectively. The γ functions are either flat (slope = .01 for bird 451) or slightly rising (slope = .17 for bird 263).

Thus, on the basis of the functions for the mean break point, a no-timing or absolute-timing alternative is not tenable, and on the basis of the standard deviation data, Poisson-timing as well as absolute-timing appear not tenable either. The slopes of the standard deviation functions are, if anything, greater than 1, rather than close to $\frac{1}{2}$, as predicted for Poisson-timing, or 0 as predicted for absolute-timing. This is reflected dramatically in the coefficient of variation data, indicating that the Weber fraction remains unchanged (or increases slightly) rather than decreases with increasing size of the timing interval.

The range over which the scalar property holds is remarkable. It is nearly two orders of magnitude in these data. In the data from Dews (1970) shown in Figure 1, it spans three orders of magnitude. Evidently, variance increases as the square of the mean out to time values of nearly an hour in the fixed-interval paradigm. This is to be contrasted with the difficulty in maintaining timing behavior for birds and rats under reinforcement schedules which reinforce spacing of individual responses at values of more than about 30 sec (Catania, 1970; Staddon, 1965).

An interesting variant of the fixed-interval procedure has been studied by Harzem (1969) in which fixed-interval values increased throughout the session. Rats were studied extensively on a schedule in which successive fixed-interval values were increased either arithmetically ($T_n = T_{n-1} + K$) or geometrically ($T_n = KT_{n-1}$, K constant), within each session. Harzem reported data from two subjects, one under the arithmetic and one under the geometric progression, and these data are replotted in Figure 6 on double log coordinates. The measure is the mean time until the second response after reinforcement. This measure is comparable to the break-point analysis performed by Schneider. The open circles are data from the animal studied with an arithmetic progressive fixed interval, in which the duration of successive intervals was incremented by 1 min. The second animal, represented by filled circles, was studied at geometrically increasing fixed-interval values with successive intervals representing an increment of 1.12 over preceding intervals. The geometric series greatly exceeded the range of the arith-

metic series. For the geometric subject, pausing after reinforcement deteriorated at FI values longer than about 50 min. Only values from the two subjects over the range of 1–50 min are shown in the figure. The best fit line has a slope of 1.016 and accounts for 94% of the variance. The subject with the arithmetic progression appears to have somewhat longer pause times at short intervals and the subject with the geometric progression appears to have somewhat shorter times for short intervals, but for the range from about 3 to 50 min, both subjects' data are well approximated by the best fit line. Variance data were not reported in this study and it is likely that the progressive character of the schedule would greatly increase variance. The progressive schedule requires that subjects hold "in memory" the rule for incrementing successive intervals, and it is remarkable that the mean pause times reflect these increments as well as they appear to do.

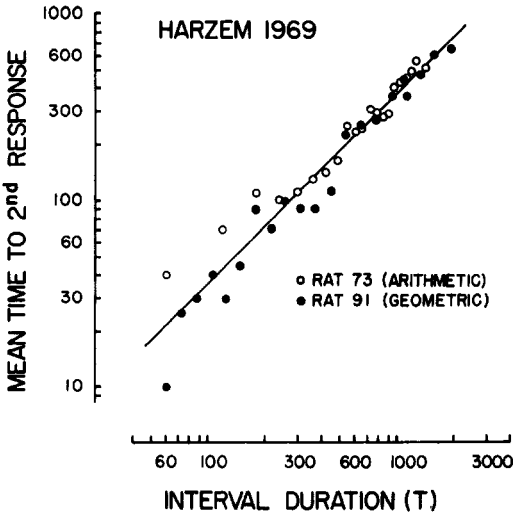

Figure 6. Mean time until the second response in each fixed interval, under progressive fixed-interval schedules for two rats. In the arithmetic progression, successive FI values within a session increase by a constant, and in the geometric progression, successive FI values increase by a factor. Both scales are logarithmic.

Fixed-Time Reinforcement: Activity

An extensive parametric study of activity of pigeons when reinforcement is delivered at fixed-time intervals noncontingent upon behavior (FT reinforcement) has recently been reported by Killeen (1975). Killeen appeared to assume something like the scalar property in his analysis, since activity data were analyzed in terms of proportions of the fixed interreinforcement time rather than absolute values of this time. Activity was measured in a chamber which contained different segments sensitive to the animals' weight so that movement was recorded as the frequency of changes from one section to another. He found that activity was generally highest about $\frac{1}{5}$ of the way through the interval and declined toward the end of the interval as the time for reinforcement delivery approached. He proposed a theoretical account of activity in which behavior within the interval is analyzed in terms of "interim" responding and "terminal" responding similar to Staddon and Simmelhag's (1971) account. Staddon and Simmelhag (1971) with pigeons, and more recently Staddon and Ayres (1975) with rats, showed that some (interim) behaviors tended to predominate in the early and middle portions of the interreinforcement interval, and other (terminal) behaviors predominated later in the interval, just prior to reinforcement delivery. With both species, interim behaviors tended to be more variable and to involve more activity (e.g., running with rats, wing flapping and pacing with pigeons), while terminal behaviors tended to be less variable, localized to the feeder area, and directly related to consummatory responses (e.g., pecking with pigeons, gnawing and digging with rats).

Expectancy theory applied to this situation approximates the obtained probabalistic transitions between successive behaviors within the interval by deterministic transitions from early, to interim, to terminal behavior at successive portions of the estimated time to reinforcement. At the outset of each interval, subjects are assumed to estimate time to reinforcement and then to initiate different rates of activity at different portions of this estimated time. Early in the interval, the expectancy ratio is close to 1, and low rates of activity occur. This early period is quite short and is intended to reflect some minimum latency for interim responding to develop at a higher rate. The early period may also involve, however, some behavior elicited by the preceding reinforcement (no-timing) rather than responding

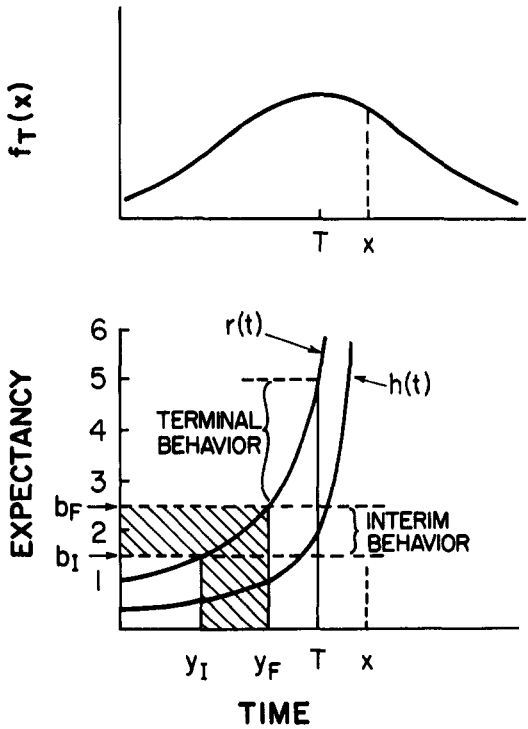

Figure 7. Estimation and expectancy for activity. The top panel shows an estimation distribution, $f_T(x)$, for the fixed interfood interval, (T), with a sample, x, indicated (as in Figure 2). The lower panel shows expectancy, $h(t)$, and expectancy ratio, $r(t)$, appropriate to this sample. Two thresholds are assumed. When the expectancy ratio crosses the interim threshold, b_I, at y_I, interim activity begins and continues until the expectancy ratio crosses the final threshold, b_F, at y_F, when terminal activity begins and continues until the end of the interval, T.

in anticipation of the forthcoming reinforcement. When expectancy rises above a threshold for interim behavior, b_I, interim activity begins, at a higher rate, and continues until expectancy rises above a second threshold, b_F, for "final" or terminal behavior, when activity is again low. Clearly, these ideas could be expanded so that different rates of activity appropriate to any different level of expectancy would be possible. The data, however, may be approximated with a single time window corresponding to a single estimate, x, with interim behavior occurring between the two thresholds, b_I and b_F.[3]

This approximation is depicted in Figure 7. The top panel shows a distribution of time estimates about the fixed-time food delivery period, T, with a sample, x, as in the fixed-interval case (Figure 2). In the lower panel, the expectancy function $h(t)$ and the expectancy ratio function $r(t)$ are shown for this sample. From the beginning of the interval until $r(t)$ reaches the threshold for interim activity, responding is assumed to occur at a low rate appropriate to the beginning of the interval, $\bar{R}_B$. Activity changes to a higher rate, $\bar{R}_I$, appropriate to interim periods while $r(t)$ is between, the interim threshold and the final threshold, that is, during the time interval between y_I and y_F when the expectancy ratio crosses these thresholds. Between y_F and the end of the interval (if $y_F < T$), responding returns to a low rate appropriate to final or terminal behavior, $\bar{R}_F$. Over successive intervals, the time window for interim responding varies with the estimate, x, (according to Equation 3). Over many intervals, the rate of activity at a given point, t, between zero and T, is an average of the rate at that time for intervals in which $t < y_I$, $(\bar{R}_B)$, those in which $y_I < t < y_F$, $(\bar{R}_I)$, and those in which $y_F < t$, $(\bar{R}_F)$. It is shown in Appendix A that if the estimates, x, are scalar in T, then the mean rate of activity function, $\bar{R}_T(t)$, is also. Then defining an activity density function,

$$g_T(t) = \frac{\bar{R}_T(t)}{\int_0^T \bar{R}_T(z)dz}, \tag{6}$$

as the proportion of total activity at time, t, it is shown that this density should be scalar in T also.

$$g_T(t) = \frac{1}{T} g(u), \tag{7}$$

where $u = t/T$. This means that the functions for the proportion of total activity should superimpose when plotted at proportional points within the interval (t/T).

Killeen has kindly sent me raw data, and

[3] A single time window for each estimate means that late initiations of interim responding should correlate with late initiations of terminal responding, which appears not to be the case (Staddon & Ayres, 1975). A more realistic theory might allow for multiple estimates of time to reinforcement during the interval, thus eliminating this correlation and producing probabilistic transitions between successive behaviors within each interval.

distributions of activity averaged over three subjects are shown in Figure 8 for three T values. These distributions nearly superimpose when plotted at proportions of the fixed-time interval. The smooth curve was obtained with a modified staircase fitting procedure assuming no activity early and late in the interval ($\bar{R}_B = \bar{R}_F = 0$). The unit timer was assumed Gaussian in form, with $\mu = 1, \gamma = \sigma = .7$, and the thresholds were set at $b_I = 1.06$, $b_F = 2.2$. The normalized rate function, $g(u)$ represents a reasonable description ($\omega^2 = .956$), but there are too many free parameters for a realistic discrimination of the power of the account. However, the superposition of these distributions, independently of their form, represents an important validation of the scalar property. These three subjects are particularly good illustrations of this property. Some others, particularly when studied at longer T values, show a tendency toward shallower activity distributions with increasing T. This is what would be expected if early responding, $\bar{R}_B$, were elicitative (no-timing) rather than anticipatory. Then the contribution of reinforcement-elicited responding early in the interval would constitute a greater proportional increment for shorter T. The result would be a greater peaking of the distributions for small T as well as a higher overall rate of activity, as Killeen finds (Killeen's values for maximum rate, A, in Table 1). However, the effects of elicitation on the moments of the activity distributions are small relative to anticipatory behavior related to T.

The scalar property, Equation 7, is distribution free, and so a robust test of this property is effected by examining the means and standard deviations of the activity distributions in Killeen's (1975) study. The mean, standard deviation, and coefficient of variation functions for eight animals are shown in Figure 9 on double log coordinates. Means are indicated by circles, standard deviations by triangles, and the coefficient of variation functions by squares. The scalar property requires slopes of unity for the mean and standard deviation and zero for their ratio. The linear functions shown are best fitting straight lines and the slope predictions are very well approximated. Slopes for the mean and standard deviation functions vary from 1.19 to 1.06 in Experiment 1A (right column) and from 1.08 to .98 in Experiment 1B (left column). The coefficient of variation slopes vary from .007 to .019 in Experiment 1A and from .003 to .017 in Experiment 1B. The proportion of variance accounted for by the mean and standard deviation fits is in all cases greater than .982.

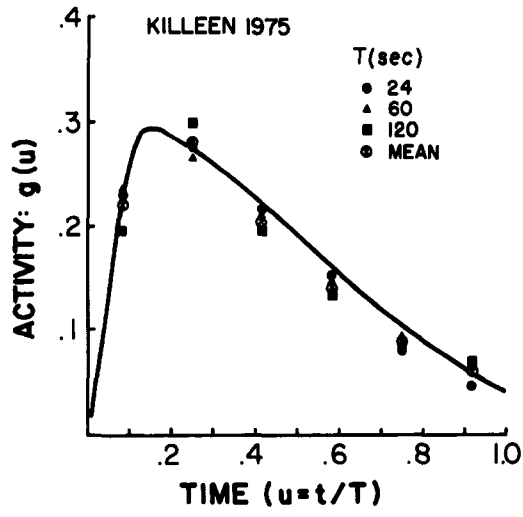

Figure 8. Normalized activity distributions, $g(u)$, averaged over three subjects at each of three different values of interfood intervals, (T). Proportion of total activity is plotted as a proportion of the interfood interval, $u = t/T$, and the smooth function is a fit of the theory to the data, assuming activity is essentially composed of interim responding.

Here also, scalar-timing provides a good description of data under strictly periodic reinforcement. For the activity data, the account is particularly compelling with respect to the constancy of the coefficient of variation. It should be remembered, of course, that the interpretation of this constant is not the same here as with the fixed-interval data. There it represents the sensitivity index for the unit timer. In the activity data, it is a derived measure for the scalar activity density distributions and may be shown to depend on the thresholds as well as on the sensitivity of the unit timer. Thus, there is no reason to expect the coefficient for the FI data ($\gamma \simeq .35$) to correspond to its value for the FT data ($\gamma \simeq 1.0$).

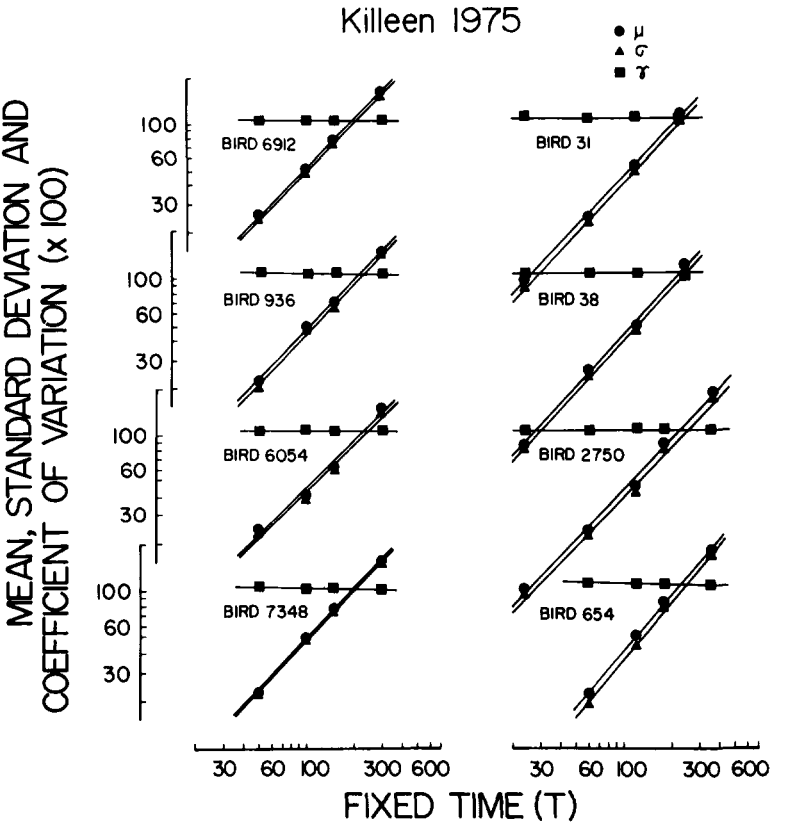

Figure 9. Mean (circles), standard deviation (triangles), and coefficient of variation (squares) of activity distributions for eight subjects. Both axes are logged and data for each subject have been shifted vertically by .6 log units.

Interval Estimation

Fixed-interval and fixed-time schedules of reinforcement are not the only, or even the most frequent, paradigms for the study of timing in animals. Much more extensive work has been devoted to the study of differential reinforcement of low rate (DRL) in which reinforcement is tied to a minimum spacing of responses in time (Kramer & Rilling, 1970). This procedure, however, results in a distribution of reinforced time intervals rather than a single value.

When the reinforcement requirement is that responses be spaced at least T sec apart for reinforcement delivery, after sufficient training subjects typically come to space their responding approximately at the minimum reinforced interresponse time. When T is changed, an appropriate change also occurs in the interresponse time (IRT) distribution, but subjects experience considerable difficulty in lengthening interresponse time beyond about 1 min for rats or even shorter times for pigeons. Evidently, reinforcement tends both to increase the frequency of appropriately spaced responding in DRL schedules and also to increase the frequency of responding rapidly, so that the two processes work against each other as T is increased. Within the range over which subjects are able to adjust their interresponse times, the result is a distribution of reinforced interresponse times which represents a decreasing proportion of the total interresponse time distribution as the timing criterion is increased. Generally, also, the distribution of reinforced interresponse times is not of a constant form with changes in T. Rather, subjects at asymptote produce an interval of reinforced IRTs in which reinforcement most frequently occurs

close to the lower bound of possible reinforced responses, and the range of the reinforced interval decreases with increases in the minimum criterion time (e.g., Malott & Cumming, 1964).

The application of scalar expectancy theory to this situation requires that subjects estimate different points within the reinforcement interval with probabilities appropriate to the probabilities with which those interresponse times have been reinforced in training. An ideal theory of this kind would specify the way in which the form of the interresponse time distribution (both reinforced and unreinforced portions) evolves as a function of training (Norman, 1966). The theory proposed here does not specify how asymptotic responding under DRL is approached. Rather, the expectancy ideas will be applied to the distribution of reinforced interresponse times as though it were an independent variable manipulated by the experimenter rather than, as is in fact the case, a dependent variable resulting from a history of reinforcement.

The problem is illustrated in Figure 10 in which three times values greater than T, τ_1, τ_2, τ_3, are assumed to have been reinforced with differing probabilities ($p_1 = .7$, $p_2 = .2$, $p_3 = .1$). The smooth solid functions, $f_\tau(x)$, are normal curves illustrating scalar estimates of these three time values. The resulting overall estimation distribution [$f_T(x)$, dashed function] is produced from the component distributions as a mixture. The mixing distribution is the probability associated with each τ value. Passing to a continuous distribution of reinforced time values, the mixture of estimates is

$$f_T(x) = \int_T^\infty f_\tau(x) f_T^*(\tau) d\tau, \qquad (8)$$

where $f_\tau(x)$ is the estimator for τ, and $f_T^*(\tau)$ is the distribution of reinforced time values (τ). Two important problems present themselves immediately in Equation 8. The first, noted above, is empirical. Both the time estimates, f, and the reinforcement distribution, f^*, are produced by the subject, thus confounding the dependent and independent variables. The analysis which follows ignores this problem and asks instead for the relation between the distribution of reinforced interresponse times and the overall distribution of interresponse times.

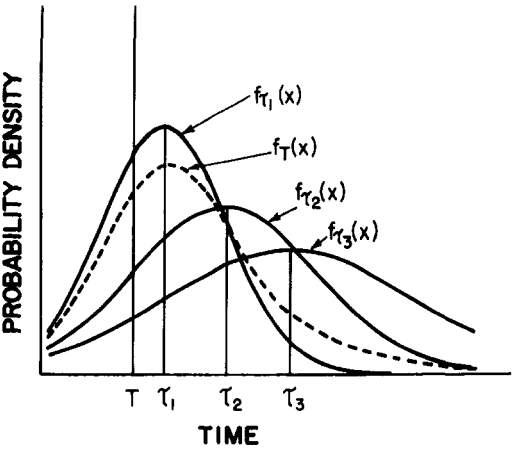

Figure 10. Interval estimation. The three solid curves represent point estimates of three time values greater than T: τ_1, τ_2, and τ_3. They are drawn assuming the scalar property, and the mixture shown as the dashed function $f_T(x)$ is a weighted sum of the three curves. The weights are the probabilities $P(\tau_i)$. In this example, $P(\tau_i) = .7, .2, .1$, for $i = 1, 2, 3$, respectively.

A second problem is a formal one. Ideally, a theory of interval estimation would obtain a closed form solution to Equation 8 for a reasonable version of the unit timer, for example normality, and a reasonable assumption about the reinforced interresponse times, for example, an exponential form. It happens that solutions of scalar mixtures are very difficult to obtain for reasonable assumptions about f_τ and f_T^*.[4] However, it is possible to relate the first two moments of the overall distributions to the first two moments of the reinforcement distributions. Substantively, this is equivalent to arguing that at asymptote, subjects' timing behavior reflects estimates of various points within the reinforcement interval, in just the proportion that these time values have been

[4] This is certainly true of the assumption of normality for the unit timer and an exponential form for the reinforcement distribution, both of which are reasonable approximations to the data. It is also true for very simple distribution forms. For example, assuming an exponential form for both the scalar kernel and the mixing distribution results in the exponential integral, $-Ei(-\tau)$ on the right side of Equation 8, which has no closed form representation. This problem is a serious one and will recur in the discussion of variable-interval schedules later.

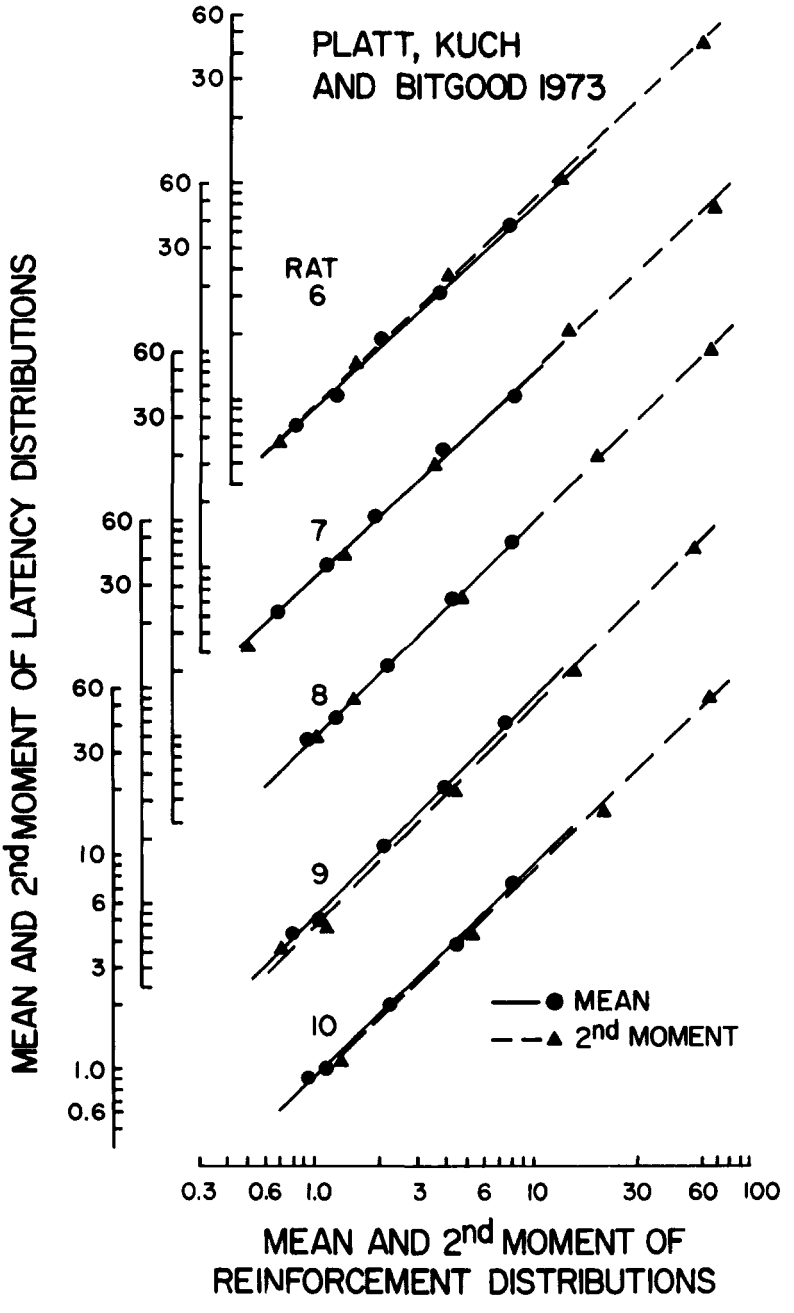

Figure 11. Mean (circles) and second moment (triangles) of overall latency distributions as a function of the mean and second moment of the reinforced latency distributions for five rats. Both axes are logged, and the data for each subject have been shifted vertically by .6 log units.

reinforced in the past. Despite the independent/dependent variable confounding in such a view, it has the advantage of showing that previous accounts which find power law relations (e.g., Catania, 1970) between the mean interresponse time and the lower limit of the reinforcement interval do so by virtue of ignoring the size and extent of the reinforced

distribution above this minimum criterion time.

The expectancy ratio rule for this situation regards subjects as sampling a time estimate from the mixture at the outset of each interresponse time, forming an expectancy ratio, and responding when the value of the ratio crosses the threshold, b. Thus, interresponse times, y, are again proportional transforms of the time estimates, x, and are governed by Equation 3. The threshold in this case, however, may be assumed to be large, as short IRTs are "punished" by nonreinforcement.

The moments of the mixture, $f_T(x)$ (Equation 8) may be obtained from the relations,

$$E(x) = E[E(x|\tau)], \tag{9a}$$

$$Var(x) = E[Var(x|\tau)] + Var[E(x|\tau)], \tag{9b}$$

(cf. Feller, 1966, p. 164). Equation 9a, in the present context, means that on average, subjects time the mean of the reinforcement distribution, not its lower limit, T. Only in case the reinforcement distribution is also scalar will timing appear to follow a proportionality rule in T. This is not generally the case, due to the difficulty described earlier that spaced responding is difficult to maintain at long values of T. This difficulty is compounded in Equation 9b, since the additive contribution of the variance of the mean of the estimation distributions decreases (along with the range of reinforced times) as T increases.

When the reinforcement distribution is not scalar, proportionality holds only between the raw moments of the overall, and reinforced, IRT distributions. These are found in Appendix B to be

$$\mu(y;T) = K\mu\mu^*(T), \text{ and}$$
$$\mu_2'(y;T) = K^2\mu_2'\mu_2^{*\prime}(T), \text{ or} \tag{10a}$$

$$\sigma^2(y;T) + \mu^2(y;T) = K^2[\sigma^2 + \mu^2] \times[\sigma^{*2}(T) + \mu^{*2}(T)], \tag{10b}$$

where $K = 1 - 1/b$ is the proportionality constant for the expectancy ratio threshold (Equation 3), and μ, σ, and $\mu^*(T)$, $\sigma^*(T)$ are the mean and standard deviation of the unit timer, and the reinforcement distributions for T, respectively.

Two parametric studies of DRL responding illustrate this analysis. In both studies, the onset of the timing interval and appropriately timed latencies exceeding the lower limit were exteroceptively cued. Platt, Kuch, and Bitgood (1973) studied rats on a modified DRL schedule in which the duration of a lever press was required to exceed T sec for reinforcement. Subjects began the timing interval with a lever press and ended it with a lever release, thus providing extensive feedback on both the beginning and ending of the timing interval. These authors published distributions which were analyzed as in Footnote 2, and data for the first two moments are presented in Figure 11. Ordinate values represent the first two moments of the overall press-release latency distributions as a function of the first two moments of the reinforcement distributions at five values of T (T = .4, 8, 1.6, 3.2, and 6.4 sec). Data for each rat have been shifted vertically by .6 log units. The ordinate and abscissa are both logarithmic and so proportionality implies functions with a slope of 1. The slope of the least-squares functions shown range from .92 (rat 6, mean function) to 1.02 (rat 9, mean function). Rats 8 and 10 have slopes equal to .99 for both moments. Proportionality as a function of the moments of the reinforcement distributions is well approximated in these data, and the proportion of variance accounted for by best fit lines exceeds .99 in all cases. This is to be contrasted with the authors' presentation of log median press durations as a function of log T, in which they report slopes ranging from .82 to .98. (In their presentations, the lowest duration, T = .4, was excluded from the analysis. Inclusion of these points would have resulted in lower slopes in all cases). It is worth emphasizing that the range of T over which the subjects were studied did not exceed 6.4 sec. This is to be contrasted with the range of FI data for rats presented earlier (Figure 6) in which interfood durations of 50 min appeared to be close to the upper limit of scalar performance.

A similar restriction in the range of DRL performance is evident in a study by Catania (1970) using pigeons. In this study also, exteroceptive feedback was provided for the beginning and end of the timing interval. The subjects were exposed to a discrete trial procedure in which key-light onset began a time interval and if the first response occurred after T sec,

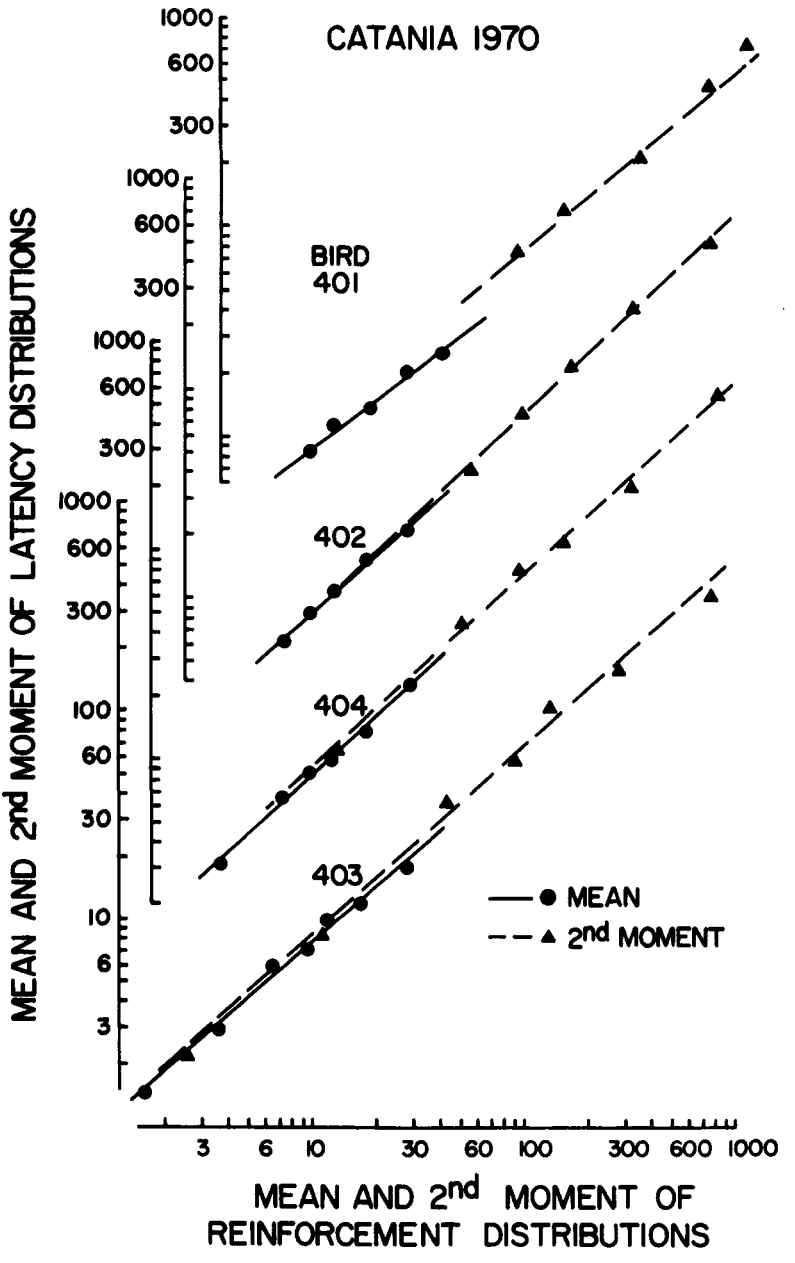

Figure 12. Mean (circles) and second moment (triangles) of overall latency distributions as a function of the mean and second moment of the reinforced latency distributions for four pigeons. Both axes are logged and the data for each subject have been shifted vertically by .6 log units.

it was reinforced with food. Subjects were studied to asymptote at a range of T values, restricted by the requirement that some substantial amount of reinforcement be obtained during the session. This resulted in a restricted range of T. The maximum T was 48 sec, with most subjects being unable to maintain responding at T values longer than 24 or 36 sec. The data have been analyzed in the same way from published distributions (Catania, 1970,

Figures 1–6).[5] The first two moments of the overall latency distributions are presented as a function of the first two moments of the reinforcement distributions in Figure 12. Again, data for each bird are displaced by .6 log units on the ordinate, and the scales are both logarithmic. The data here show deviations from proportionality not observed in the rat data. Bird 401, the most deviant, has slopes of .76 and .79 for the mean and second moment functions, while the slopes of the remaining functions vary from .88 to .95. Again, also, variance accounted for by linearity is very high ($\omega^2 = .98$).

The theoretical account is less closely approximated with the pigeon data than with the rat data. A possible contributing factor is the tendency of birds to peck signals associated with food, independently of the DRL contingency (Brown & Jenkins, 1968). In the paradigm studied by Catania, signal onset began the timing interval and an appropriately delayed response produced food. Thus, the signal–food pairing may have contributed to a tendency toward short latency (autoshaped) responding. The slopes obtained here are to be contrasted, however, with those published by the author for log mean responding versus log T, which range from .73 to .88. (Those functions also excluded the shortest T value, which would have lowered the slopes still further.)

The value of the sensitivity parameter in these data is $\gamma \simeq .5$, which is not very different from that for the FI data ($\gamma \simeq .35$). However, estimates of the threshold are larger here ($b \simeq 5$), than under FI ($b = 2.2$).

This completes the analysis of estimation procedures in animal timing. The fixed-interval and fixed-time procedures are perhaps the most convincing for the scalar expectancy view, since DRL procedures are complicated by the interval feature of the time estimation process, which means that subjects generate both the reinforcement distribution and the latency distribution. Also, birds, at least, appear to deviate somewhat from the scalar rule although the deviations are not large. One possible source for this deviation is worth noting. If one regards the response itself as requiring a minimum latency with a mean and variance, which adds to the time estimation distribution but is unaffected by the duration of the time being estimated, then the result is linearity rather than proportionality in the mean and standard deviation functions. This is the so-called "generalized Weber law" (Church, Getty, & Lerner, 1976; Getty, 1975; Treisman, 1963). Linearity tends to reduce the slope of the best fit double logarithmic plot. Addition of a motor component would be observable only in latency data (e.g., avoidance, Figure 1, and DRL) and especially at low values of T. Thus, it is not surprising that the FI data, which span a very broad range of T values, show no deviation from the scalar rule.

Summary and Discussion of the Theory for Estimation

Timing

The core of the timing proposal is that behavior under time-correlated reinforcement schedules reflects estimates, on a trial-by-trial basis, of time to the next reinforcement presentation. Timing data argue that the way in which variance in time estimates increases with the value being timed is scalar. The evidence adduced here is drawn from appetitive reinforcement procedures and is strengthened by previous accounts of scalar-timing in avoidance (Gibbon, 1971, 1972). The account is a parsimonious one in the sense that it involves but one application of that distribution to a given timing interval. The "unit timer" is a convenient (though arbitrary) description of this form, since the subjective time units are unspecified here, and transformations are possible either for shorter or longer time values. From human psychophysical data it seems likely, however, that the range over which the scalar process applies is limited at the lower end by about ½ sec. It is convenient therefore to think of the unit timer as appropriate to this lower limit. The time values studied in the data explored here, then, multiply this unit by the appropriate scale factor. It is convenient also to think of the form of the unit timer as

[5] Distributions were not used that contained a "dump" category exceeding 10% of the distribution, or that contained fewer than 5% reinforced latencies. Otherwise, estimates were obtained by the photographic method outlined in Footnote 2.

approximately normal. For most of the predictions of the theory in estimation, this form is not critical, although latency distributions for moderate time values (e.g., Platt, et al., 1973) are roughly bell shaped. A symmetric normal form reduces the two timing parameters (μ,σ) to the single sensitivity index, $\gamma = \sigma$, the coefficient of variation.

Expectancy: H and h(t)

When reinforcement is strictly periodic in time, the behavioral indexes of timing occur in the interval between reinforcements. The expectancy function (Equation 1) represents the means by which the theory translates estimates of reinforcement time into behavior considerably in advance of reinforcement delivery. Expectancy may be thought of as an estimate of instantaneous rate of reinforcement. Reinforcers generate an internal excitatory strength, H, which is spread back over the estimated time before delivery. H is a motivational parameter which should vary with amount and probability of reinforcement (incentive) and with deprivation (drive). Colloquially speaking, H represents the total amount of "hope" a single reinforcer may support, and the instantaneous rate of expectancy is the total value of a reinforcer weighted by its "relative immediacy" (Chung & Herrnstein, 1967). Alternatively, one may think of the expectancy function as the total value of the reinforcer weighted by the probability density of its occurrence, where that probability density is uniform over the remaining time before reinforcement delivery. In this context, $(1/H)h(t)$ is the probability density associated with the lower bound of the family of uniform distributions generated as t moves from the beginning of the interval to its estimated completion.

For any fixed t,

$$H = \int_t^x h(t)dz$$

is constant.

One might readily conceive of timing behavior as reflecting expectancy of reinforcement alone, without reference to a base level, $h(0)$. It is the expectancy ratio that forces the interpretation of timing as a discrimination task. A simple alternative would regard timing behavior as emerging when the expectancy function itself reaches the threshold, b. For the hyperbolic expectancy function (Equation 1) this results in linearity rather than proportionality in Equation 3, $y = x - H/b$. A negative intercept is thus implied for the mean timing functions, which is contrary to the proportionality evident in the data (Figures 5, 6, 9, and 11). Also, motivational effects should be observable on this construction, and evidence on this point is discussed below.

Simple timing as a function of simple expectancy might still be tenable, however, if the expectancy function were not hyperbolic. A straightforward alternative with some face validity is the integral of the uniform spread of expectancy back over the estimated time to reinforcement. On this view expectancy accumulates over the timing interval and subjects respond when the integral exceeds the threshold. Equation 1 then becomes

$$h(t) = \frac{Ht}{x}, \tag{1a}$$

and Equation 3 becomes

$$y = \frac{b}{H}x. \tag{3a}$$

Proportionality is preserved in Equation 3a, and so the scalar property holds on this construction. However, motivational effects again should be observable here, and this point is discussed next.

Expectancy Ratio: r(t)

The central issue for alternatives to the expectancy ratio construction is whether motivational effects may be observed in timing. Expectancy theory requires no change in timing with changes in H, and this implication appears to run counter to intuitive and traditional concepts of the role of motivation in learning. Evidence is examined below. The data are unequivocal with respect to deprivation and somewhat less clear with respect to magnitude and probability of reinforcement.

Deprivation. Weiss and Moore (1956) studied FI performance of two groups of rats under 4-hour and 22-hour deprivation conditions.

Subjects were trained on a 3-min FI schedule, and differential responding over the timing interval (FI scallop) was obtained. They found that subjects under the high deprivation conditions responded about twice as often overall as those under the low deprivation condition, but the distribution of responses over the interval was the same for both groups. Responding showed an approximately linear increment over the FI interval, and the slopes for the two groups were nearly identical. That is, the proportion of responses in a given proportion of the interval was the same for both deprivation levels. This is consistent with an increment in H with deprivation, but no change in timing because of the comparison of local with overall expectancy.

Similar effects have been found under DRL. Conrad, Sidman, and Herrnstein (1958) studied rats under DRL schedules at deprivation levels ranging from 9 hours to 69.5 hours, and studied a monkey under a similar deprivation range. They found no change in the timing distributions unless subjects were satiated or under virtually no deprivation, when response rates were very low. Mechner and Guevrekian (1962) studied rats under a 2-lever DRL procedure in which a response on lever A initiated a $T = 5$ sec timing interval, and a response on lever B was reinforced only when the A–B latency exceeded T. Deprivation levels ranged from 4 to 56 hours. They found that increasing deprivation increased the frequency with which A–B pairs were initiated, but had no effect on the temporal spacing or efficiency of the timing performance. Migler and Brady (1964) extended this finding to a drive interaction situation. They found that rats in the 2-lever DRL initiated A–B pairs less frequently in the presence of a preshock (CER) warning signal than in its absence, but their timing performance was unaffected by the signal.

Reinforcement magnitude. Possible effects of reinforcement magnitude on timing behavior are less clear. Beer and Trumble (1965) studied rats on a 2-lever DRL task using exteroceptively cued, different amounts of reinforcement. The different amounts were programmed in successive 5-min portions of the session, each with its own auditory signal. The results showed a decrease in mean A–B latencies with increasing reinforcement magnitude—contrary to the expectancy ratio requirement of independence. However, the authors noted that no change in timing was evident unless the different magnitudes were presented within the same session. Kramer and Rilling (1970), in reviewing this and related work in DRL, concluded that reinforcement magnitude effects generally were found only when magnitudes were mixed within the same session. The effect thus appears to be a context phenomenon. The intriguing possibility arises in the expectancy account that the reference level, $h(0)$, for timing on any trial is an average over the different H values (magnitudes) in a session. Expectancy for anticipation of a large reward would then exceed the average reference level earlier in the timing interval, and conversely for anticipation of a small reward. Temporal spacing of responding should then vary inversely with reinforcement magnitude, as Beer and Trumble found.

Early work on reinforcement magnitude in FI schedules with pigeons (Keesey & Kling, 1961) found no effect on timing after sufficient training when different reinforcement amounts were cued with different exteroceptive stimuli and presented in alternate sessions. However, later work using different milk concentrations with rats (Lowe, Davey, & Harzem, 1974), and different feeder durations with pigeons (Staddon, 1970) has found increased postreinforcement pausing *following* greater reinforcement magnitudes. Again the effect is contextual, with different magnitudes presented within each session. However, in this case reinforcement magnitudes were not differentially cued, and the effect is thus not an anticipatory (incentive) one in the usual sense. These investigators identified the lengthening of pause time with augmented inhibitory control early in the interval for greater preceding reinforcer magnitudes. However, in an expectancy framework, one might also argue for an effect of the preceding reinforcer magnitude on the reference level, with a larger preceding magnitude generating a larger $h(0)$ for the current trial. Since the forthcoming magnitude was not cued, local expectancy for the current trial should reflect the average magnitude, with the result that $h(t)$ should exceed $h(0)$ somewhat later than after a preceding reinforcer of a

small magnitude. The effect of "remembering" the preceding reinforcer magnitude would then be just the reverse of the effect of anticipating a forthcoming reinforcer magnitude. Anticipating an average magnitude with reference to a remembered large magnitude should lengthen pausing, while anticipating a large magnitude with reference to a remembered average magnitude should shorten pausing. Arguments of this sort are highly speculative, but they do illustrate interpretations of contextual effects within the framework of the theory without compromising the independence of timing and motivation.

Reinforcement probability. The applications of expectancy theory to follow construe reinforcement probability as a motivational manipulation. Subjects are viewed as averaging omission of reinforcement with its presentation and forming an expectancy of Hp^*, where p^* is reinforcement probability. In simple timing schedules, however, local expectancy should still cancel in ratio to overall expectancy, and so it is important to examine evidence of possible effects of p^* in periodic reinforcement schedules.

Perhaps the best evidence on this score comes from a study by Zeiler (1972). He studied pigeons on FI schedules with p^* values ranging from .07 to 1.0. Under one procedure, there was a time-out signal on the key when reinforcement was not presented, and both reinforcement and nonreinforcement occasions were followed by a 10-sec blackout stimulus before the next FI interval was initiated. Reinforcement probability reduced overall responding, particularly for $p^* \leqslant .3$. But there was no change in pausing at the outset of the FI interval until $p^* = .07$. At this value subjects occasionally failed to respond for most or all of the interval, whereas when substantial responding did occur, timing appeared to be about the same as at higher p^* values. This finding is similar to that of Weiss and Moore (1956) for changing deprivation level. Decreasing motivation decreases overall responding but does not change timing efficiency.

This demonstration depends heavily upon presentation of a relatively long blackout in lieu of, or along with, reinforcement delivery at the end of the timing interval. When a blackout is not interpolated between FI intervals (Zeiler, 1972), or when it is short (Staddon & Innis, 1969), a decrease in pausing is observed following reinforcement omission, which is similar to "frustration" effects (e.g., Amsel, 1958; Scull, Davies, & Amsel, 1970). Decreased pausing after nonreinforcement, when it is found, appears to be a context effect which depends on the stimulus presented in lieu of reinforcement (Staddon, 1972a, 1972b; Staddon & Innis, 1969; Zeiler, 1972), and it does not appear to be an anticipatory (incentive) effect. In one of their procedures Staddon and Innis (1969) differentially cued probabilistic ($p^* = .5$) and regular ($p^* = 1.0$) reinforcement in different FI 2-min cycles which alternated during the session. Pausing at the outset of the FI interval was the same in both cues when the timing interval began with a preceding reinforcement, while decreased pausing was observed following a brief blackout presented when reinforcement was omitted on the $p^* = .5$ schedule.

Contextual effects of this sort, from both reinforcement magnitude and probability investigations, remain a challenge to the theory in its present form and may force modifications in interesting ways. However, the majority of the data are consonant with the proposition that motivational variables affect overall response strength without affecting temporal discrimination. This independence may be more general than supposed, as Dinsmoor (1952) found that drive level affected overall response rates on an S+, S− discrimination but that relative rates remained constant. Loosely speaking, motivation changes the willingness with which subjects engage in a timing task, but not the precision with which it is executed. In the light of these considerations, the expectancy ratio is more appropriate than simple expectancy as a criterion measure controlling the emergence of timing behavior.

The alternative form for the expectancy function (Equation 1a) is not meaningful when taken in ratio to the reference level (H/x), since then timing would be independent of T. While alternative reference levels might be devised, this form (Equation 1a) for expectancy is not appropriate in later applications of the theory in which discriminations are made between temporally distant reinforcement alternatives at time $t = 0$.

Overall Expectancy: $h(0)$

The reference baseline against which the expectancy ratio evaluates current levels was taken to be $h(0)$, the expectancy level at the start of the timing interval. It is an estimate of overall or average expectancy in the sense that with a symmetric estimation process $E(x) = T$, and so for the mean estimate, $h(0)$ is the motivational value of the reinforcer weighted by the overall rate of reinforcement. Overall expectancy is not the only reference level against which subjects might evaluate current, temporally discriminated expectancy. An alternative which also preserves the scalar property and has some face validity is worth consideration. Suppose a subject on an FI schedule has not attended to the time (t) since the last reinforcement presentation. Such inattention might be most appropriate early in training, or possibly also later in training when, for whatever reason, the beginning of the interval was unnoticed or "unrecorded." On such trials, samples of the time to the next reinforcer may be thought of as (independent) samples of $T - t$, taken with uniform probability over the interval between reinforcers. A symmetric scalar estimation process based on these samples would find on the average that the expected time to reinforcement was $T/2$, since this is the mean of a uniform distribution over the interval. A new reference level might then be defined as an "undiscriminated" expectancy appropriate to $T/2$—say, $h'(0) = H/x'$, where x' is an estimate of $T/2$. On average, the new reference level is twice the old, but it may be shown that the scalar property for the mean and standard deviation of the timing distributions is preserved. The interpretation of the proportionality constant for the mean of y as a function of T is unchanged, while the interpretation of the proportionality constant for the standard deviation function differs using the new reference level. The formal development for this modification of the theory is somewhat more complex than the present analysis and will not be pursued here since the data do not discriminate between the alternatives. However, it is worth noting that, using the higher reference level, $h(t)$ is less than $h'(0)$ $[r(t) < 1]$ early in the timing interval, and $h(t)$ is greater than $h'(0)$ $[r(t) > 1]$ late in the interval—which may correspond to inhibitory and excitatory temporal control.

Expectancy Ratio Threshold: b

Timing behavior emerges at the point within the timing interval at which the expectancy ratio, $r(t)$, reaches a threshold value, b. The threshold is assumed to be constant for a given experimental setting and is a response metric parameter which reflects the "bias" between the response alternatives involved in the discrimination. In FI schedules the response alternatives are pausing (or responding at a low rate) early in the interval versus responding at the high terminal rate late in the interval. The threshold reflects a balance between these alternatives under the contingency constraint that responding is required for reinforcement delivery at the end of the interval. Changes in the nature of the response or in the contingency imposed at the end of the interval would be expected to change b. Increasing the response requirement for reinforcement at the end of the FI does increase pausing somewhat (Shull, Guilkey, & Witty, 1972), though the effect is not large.

Under noncontingent periodic reinforcement (FT reinforcement) different b values were assumed for interim and terminal behavior. Without differential response–reinforcement contingencies, b values should reflect the "natural" or "prepared" (Seligman, 1970) properties of different behavior, in the context of a given kind of reinforcer (Jenkins, 1973; Moore, 1973), and a given proximity to reinforcement (Staddon & Simmelhag, 1971). For pigeons, terminal behavior under FT reinforcement is most frequently pecking (Staddon & Simmelhag, 1971), and so it is not surprising that the obtained threshold for this behavior ($b_F = 2.5$) is only slightly larger than that for FI reinforcement when the same response, pecking, is required for reinforcement ($b = 2.2$). Under DRL schedules, in contrast, a strong negative contingency is imposed on responding early in the interval, since early responding postpones reinforcement. This constraint weights the balance between pausing and responding more heavily in favor of pausing, and obtained b values are larger ($b \simeq 5$). Since

pecking is a preferred terminal behavior for pigeons under noncontingent FT reinforcement, one might expect some improvement in DRL performance (higher b values and larger pause times) using a response less likely to be evoked "naturally" at intermediate expectancy levels. Pigeons' DRL performance does appear to be improved somewhat when the response requirement is treadle pressing (Hemmes, 1975; but see also Richardson & Clark, 1976).

In periodic reinforcement schedules the response alternatives are asymmetric in two ways. The nature of the alternatives—pausing *versus* responding of one or another sort—is qualitatively different, and the contingencies imposed for the two alternatives generally differ also. Pausing for very long periods delays reinforcement under FI and DRL, while early responding may result in some cost in effort under FI and results in substantial cost in reinforcement frequency under DRL. Even under noncontingent FT reinforcement, it may be argued that terminal behavior in the vicinity of the food magazine in some sense differentially "prepares" the subject for consumption of the reinforcer in ways that interim behavior or pausing does not. These asymmetries of quality or contingency in the response metric are reflected in asymmetric threshold or bias values ($b \neq 1$). In the discrimination and choice paradigms treated next, the response alternatives are generally symmetric so that choice between these alternatives is primarily a function of temporal discrimination and reinforcement value.

Discrimination

Intuitively, it would seem appropriate that the expectancy ratio comparator combined with scalar-timing should result in Weber's law for the discriminability of time intervals. In fact, this turns out to be correct, but the relationship is not as immediate as appears on its face. In particular, when a biased discrimination is considered, Weber's law requires both the ratio comparator and the scalar assumption and alternative decision rules or alternative timing processes produce discernible deviations from Weber's law.

Possibly the earliest description of the relation between the scalar hypothesis and Weber's law was proposed by Thurstone (1927b). In describing a hypothetical case for which he argued that Weber's law held but Fechner's law did not, he presented a table (Table II) of means and standard deviations which implied constant discriminability between adjacent pairs of stimuli. Possibly because of variations in the second decimal place of the tabled values, Thurstone evidently did not note the constant ratio ($\gamma \simeq .2$). Later mathematical treatments of animal timing (Gibbon, 1971; Norman, 1966) have assumed the scalar rule and described it as a "Weber-like" process, without specifying the relation between this assumption and Weber's law.

Similarly, theories of human time discrimination, in particular Treisman's extensive study (1963), have also assumed a constant Weber fraction in estimation to be equivalent to constant discriminability, although without specifying explicitly the response rule which would relate estimation to discrimination. In studies of human timing, a number of investigators have found Weber's law not to hold (e.g., Allan & Kristofferson, 1974; Creelman, 1962), although Treisman's study (1963) found strong support for the "generalized Weber's law" (linearity rather than proportionality of ΔT to T). Treisman's theoretical accounts (1963, 1964) offer an ingenious proposal about timing which allows for results intermediate between proportionality and Weber's law, and the "square root law" implied by the Poisson process (Figure 4). He proposed that counts on an internal accumulator are collected up to some criterion number, N. For a neural mechanism these counts would be produced by one or several Poisson sources. However, successive intercount intervals are highly correlated, with the result that the estimate,

$$x = \sum_{i}^{N} t_i,$$

with the strong correlation is approximately $x \simeq Nt$. Therefore, changing the criterion proportionally with the duration being timed results in the scale transform relation which I have called scalar-timing. A possible difficulty with this account is that the distribution of the

t_i for an accurate representation of neural processes ought to be exponential, and the data, in animal timing at least, are approximately normal in form.[6] An important difference between studies of human and animal timing is that virtually all of the human experimentation and theorizing concerns very short time intervals, generally in the millisecond range. The range over which animal timing appears to conform to the scalar rule begins around .5 sec, and the best evidence for the theory has been collected at ranges up to 50 min or so.

In work on animal timing, psychophysical procedures for the study of discrimination are rare. Kinchla (1970) studied duration discrimination of pigeons over a small range of time values (2 to 4 sec). This study focused on delineating a procedure with animals analogous to the yes–no technique in human psychophysics and did not contain sufficient depth in the range of T to allow assessment of Weber's law. A recent study by Church, Getty, and Lerner (1976) specifically attacked the question of whether Weber's law holds for time discrimination in rats by studying a modified yes–no procedure with "standard" and "comparison" durations in which the comparison was continuously adjusted to obtain 75% correct detections. They found Weber's law was a better description of their difference limen results than the square root law, although their data suggested some deviations from proportionality in the direction of the square root law.

Possibly the most extensive parametric study of time discrimination in a psychophysical yes–no procedure was conducted by Stubbs (1968) using pigeons. His data provide very strong support for Weber's law and will be analyzed after the treatment of the theoretical account below.

Weber's Law

Weber's law in a psychophysical context is usually characterized as constant discriminability with a constant ratio of an increment in stimulation to a standard stimulus. If we allow "discriminability" to take on any arbitrary level, the result is that the psychometric functions relating discriminability to the two

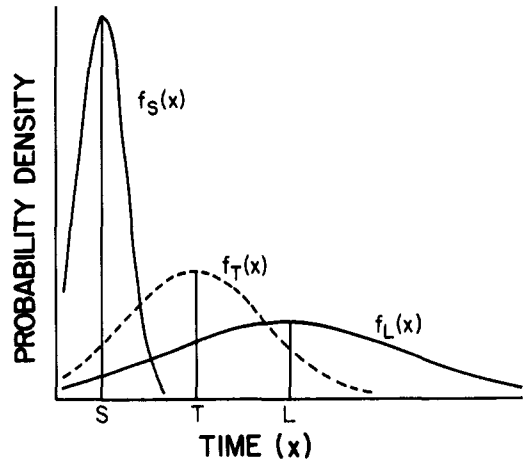

Figure 13. Distributions of perceived time values, x, associated with exposure to short (S) and long (L) stimulus durations are indicated by the solid curves. These distributions are related by the scalar rule. A distribution associated with an intermediate value, T, is shown as the dashed function $f_T(x)$.

stimulus values being compared should superimpose when plotted as a function of the ratio of the stimulus values. This is substantively the argument proposed by Luce and Galanter (1963) for the general form of Weber's law.[7]

Consider the solid curve distributions shown in Figure 13 for a "short" duration, S, and a "long" duration, L. They are intended to represent the distribution of experienced or perceived time values when subjects observe stimuli with durations S or L in a yes–no paradigm over long training. These distributions are related by the scalar rule and so their standard deviations are in the ratio L/S. The dashed distribution centered on T represents perceived time values for a stimulus of intermediate duration T, $S \leqslant T \leqslant L$, considered later. In the yes–no procedure, on any trial, either the short or the long time interval is presented and gives rise to a sample

[6] Possibly, this difficulty may be surmounted by assuming Poisson sources which register only every kth count—with a resulting Gamma distribution of the t_i. However, later applications of the scalar process show that k must be low to accommodate moderate γ values of the unit timer.

[7] This general form of Weber's law is not to be confused with the so-called "generalized Weber's law" in which $\Delta S = K_1(S + K_2)$, K_1, K_2 constant, referred to earlier.

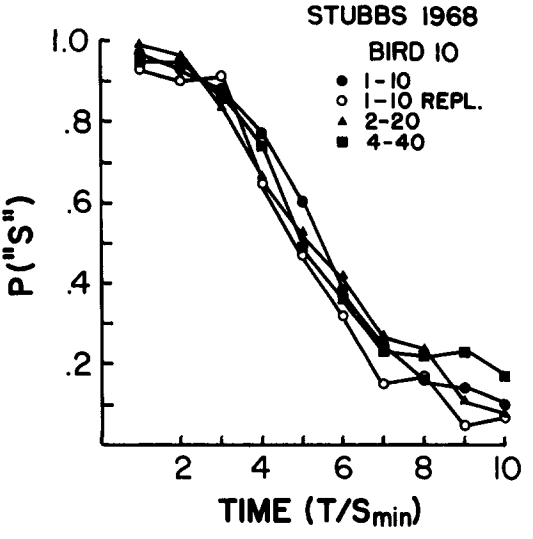

Figure 14. Probability of a short report, P("S"), at successive values of training stimuli ranging from 1 to 10 sec (filled circles), from 2 to 20 sec (filled triangles), from 4 to 40 sec (filled squares) and a replication (repl.) of 1 to 10 sec (open circles). The cutoff for reinforcement for reporting short or long in each case was located at the midpoint of the range. Abscissa values are the duration of each stimulus, T, in a given range, divided by the minimum duration in the range (S min).

perceived time value, x. The application of the expectancy ideas to this situation characterizes subjects as comparing expectancy of reinforcement for a report of short ("S") with expectancy of reinforcement for a report of long ("L") given the sample value, x. We assume for the moment that reinforcement (S^*) is only forthcoming for correct detections and correct rejections and that no reinforcement is delivered for a "miss" or a "false alarm." Expectancies in the previous analysis were formed by dividing the total amount of expectancy, H, by the time to reinforcement based on an estimate of that time. In the present case, time to reinforcement is simply the decision and response time to make a choice of a short or a long report after the stimulus duration, x, has been sampled. Denoting this time by RT, expectancy of reinforcement is

$$h_{"J"}(x) = \frac{1}{RT} HP(S^*|x,"J"), \quad J = S, L. \tag{11}$$

RT should be the same for both response alternatives, and so variation in expectancy depends on the motivational parameter, H, which must be weighted by the probability of reinforcement for each report given x. The probability of reinforcement given a short or long response is constructed from the posterior probability that x was generated by the short or long stimulus duration, and the probability, $P(S^*|J,"J")$, the probability of reinforcement given a correct detection. Calling this probability $p_J{}^*$, reinforcement probability is then

$$P(S^*|x,"J") = P(J|x)p_J{}^*, \quad J = S, L. \tag{12}$$

In the usual manner for signal detection analysis we may obtain the posterior probability of x being generated by J as

$$P(J|x) = \frac{P(x|J)P(J)}{P(x|S)P(S) + P(x|L)P(L)}, \quad J = S, L. \tag{13}$$

Now the expectancy ratio for a report "S" is formed from $h_{"S"}/h_{"L"}$, and substituting Equations 12 and 13 into Equation 11, the familiar likelihood ratio emerges,

$$r_{S,L}(x) = \left(\frac{P(S)p_S{}^*}{P(L)p_L{}^*}\right)\frac{f_S(x)}{f_L(x)} = \left(\frac{1}{\beta}\right)l_{S,L}(x), \tag{14}$$

where $\beta = (P(L)p_L{}^*)/(P(S)p_S{}^*)$ is (the inverse of) the usual likelihood ratio criterion with zero payoff for errors and probabilistic payoff for correct detections.[8] The decision rule requires the response "S" when $r_{S,L}(x) > b$ and "L" otherwise. Thus, short reports occur whenever

$$l_{S,L}(x) > b\beta. \tag{15}$$

The criterion rule (Equation 15) is simply the standard likelihood ratio criterion rule for the yes–no situation with zero payoff for errors and the possibility of response bias. Now, the likelihood ratio is distributed as $f_J(x_J)$ according as x_J is generated by $J = S, L$. The psychometric function for correct detections,

[8] The inverse is used for convenience as data here and later are presented as P("S") rather than P("L").

$P(\text{Corr.})$, is therefore

$$P(\text{Corr.}) = P(S)\int_{x_S:\, l_{S,L}(x_S) > b\beta} f_S(x_S)dx_S + P(L)\int_{x_L:\, l_{S,L}(x_L) < b\beta} f_L(x_L)dx_L. \quad (16)$$

Weber's law requires that the integrals on the right of Equation 16 depend only on the ratio S/L. This is readily shown to be a consequence of the scalar hypothesis for these densities. Consider the first integral for x_S generated by S. Transform x_S to the corresponding variate for the unit timer by $u = x_S/S$. Since $f_S(x)dx_S = f(u)du$, independent of S, it remains to find the region of integration, $l_{S,L}(x_S) > b\beta$ for the unit timer variate. The scalar property again provides the simple result

$$l_{S,L}(x_S) = \left(\frac{L}{S}\right)\frac{f(u)}{f\left(u\frac{S}{L}\right)},$$

so that $l_{S,L}(x_S) > b\beta$ is equivalent to

$$\frac{f(u)}{f\left(u\frac{S}{L}\right)} > \left(\frac{S}{L}\right)b\beta. \quad (17)$$

The region of integration for the unit timer variate, u, is therefore independent of the absolute values of L and S and varies only with their ratio. A symmetrical argument shows a similar result for the second integral in Equation 16. Therefore, psychometric functions obtained for different multiples of S, L pairs should superimpose, which is Weber's law.

Unfortunately, the experiment required to definitively confirm or disconfirm Weber's law consists of obtaining several psychometric functions, and this does not appear to have been done either in human or animal work. However, Stubbs (1968) has provided extensive parametric data on a yes–no procedure similar to that analyzed above, except that instead of single-point intervals, S, L, he used a mixture of durations for which subjects were reinforced for reporting "short" and another mixture of durations for which subjects were reinforced for reporting "long." In his experiment, pigeons were exposed to durations of 1, 2, 3, 4, 5 sec; or 6, 7, 8, 9, 10 sec by illumination of a center key. At the end of the center key illumination interval two side keys were illuminated. If the center key duration had been less than or equal to 5 sec, subjects were reinforced for a response on the left key ("S"); if it had been greater than or equal to 6 sec, subjects were reinforced for a response on the right key ("L"). After training to asymptote with equal probability of any 1 one of the 10 stimulus values, all values were multiplied by 2. Thus, the short distribution consisted of 2, 4, 6, 8, and 10 sec, and so forth. After responding stabilized on this range, the original values were multiplied by 4 (4–40 sec) for an additional determination, and then subjects were returned to the 1–10 sec range. Weber's law for this situation consists in the finding that the probability of a short report at a stimulus value, T, is the same as the probability of a short report at stimulus value KT, when the range of values is multiplied by K. In Appendix C, the preceding analysis is generalized to the mixture situation and it is shown that Weber's law holds here also if the underlying distributions are scalar. Data from one subject studied under the three ranges are replotted in Figure 14. The probability of a short report is shown as a function of the ratio of stimulus duration values to the minimum value (S min). That is, the value 1 represents 1 sec for the range from 1 to 10, 2 sec for the range from 2 to 20, and so on. Clearly, superposition for these data is confirmed, and the indifference point is appropriately near 5.5. Two other subjects studied in this paradigm show similar superposition.

The form of the psychometric function (Figure 14) depends on the form of the underlying distributions (Figure 13). Mixtures for f_S and f_L are difficult to approximate because of severe skew toward the short values of the S and L ranges, induced by the scalar property. Therefore, the theoretical development will not be pursued further here.[9] However, it may be shown that a reasonable estimate of the sensitivity index for the yes–no discrimination paradigm is in the neighborhood of $\gamma \simeq .7$, somewhat higher than that for the estimation

[9] Interested readers may write to me for the derivation of the 2-stimulus psychometric function assuming Gaussian underlying distributions. The mixture situation is treated in a manuscript in preparation.

applications ($\gamma \simeq .35$–$.5$). These values contrast with the considerably smaller estimates of the Weber fraction obtained from human time estimation work (e.g., .05–.1 in Treisman, 1963). Possibly this difference reflects additional variance for animals in "remembering" time values (estimation) or likelihood values (discrimination).

Superposition of the animal time discrimination functions independently of their form provides strong support for Weber's law. This is in contrast to deviations from Weber's law well documented for time values in the millisecond range for human time discrimination. Possibly, such discrepancies would arise in animal work as well, were the paradigms susceptible to the study of these low ranges. However, here, as in the applications to follow, animal timing is studied at relatively large values of the intervals being discriminated, and the results support a scalar estimation process and a ratio comparator as the fundamental underlying constructs.

Choice

A very large literature has dealt with choice in animals when two response alternatives are associated with different delays and magnitudes of reinforcement. Much of this work has involved variable-interval (VI) schedules of reinforcement, and average choice responding is reported as a function of the average reinforcement rates associated with the VI alternatives (e.g., Herrnstein, 1970). Except for some qualitative observations, the present account will be restricted to cases in which choices are made between fixed reinforcement delays. This constraint is forced in part by the theoretical difficulties in analyzing mixtures when the mixing distribution has a broad range, and also in part by considerable diversity among laboratories in the manner of constructing variable schedules.

Two paradigms have been used extensively to study choice in animals. Both involve two manipulanda, generally keys with pigeons, which are concurrently available to the subject. In concurrent schedules, responding on either alternative occasionally results in an amount and delay of reinforcement which is different for each alternative. In concurrent-chain schedules, both manipulanda are available only during an "initial link" choice period. Responding on either alternative occasionally results in one of two stimulus changes, correlated with one of two "terminal link" schedules of reinforcement on each manipulandum. Responding in the initial link reflects preference for the schedule alternatives in the terminal links.

Concurrent Schedules

Much of the work on concurrent choice, and quantitative accounts thereof, has been directed toward understanding the choice between two different VI schedules of reinforcement associated with the two response alternatives. However, several studies of concurrent reinforcement of two different DRL values are available. Subjects are only reinforced for spacing their responses close to two time values. The DRL values have been limited to a small range ($T + \Delta T$) so that they may be regarded for the purpose of this analysis as involving point estimates of the midpoint of this range on each alternative. For example, Staddon (1968) studied pigeons under two DRL values reinforced on a single response key. Shimp and his colleagues (Moffit & Shimp, 1971; Shimp, 1968, 1969) have demonstrated that concurrent choice for two DRL values with a single key is virtually identical to choice for two DRL values when each is associated with a different key. Shimp argues persuasively that under concurrent DRL with one key, subjects may be regarded as choosing after each response to "attempt" either the short or long reinforced IRT. The resulting interresponse time distributions are bimodal, with modes close to the two reinforced DRL values. An example may be seen in Figure 15 for one subject in Staddon's study. IRTs falling within the (hatched) DRL time windows were reinforced with different frequencies. The smooth curves are normal densities with means and standard deviations adjusted to provide a fit to the data points.[10] In this example, approximately 70%

[10] The functions have been adjusted so that their areas correspond to the probability of short and long responses. The fitting procedure resulted in standard deviations which were not in the ratio of the means.

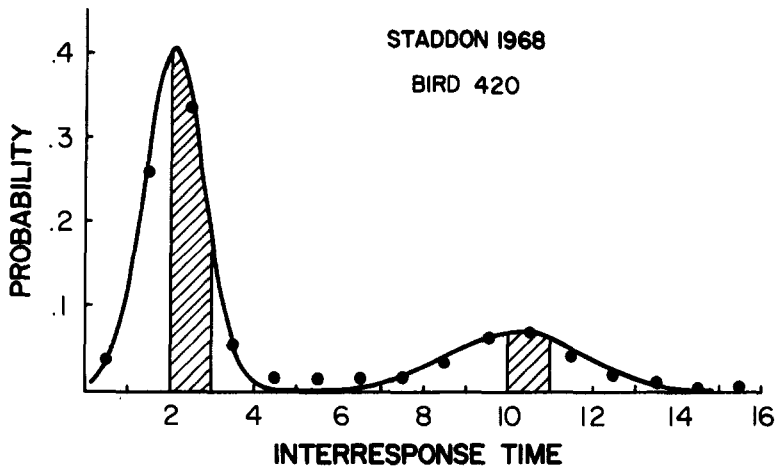

Figure 15. Interresponse time distributions for a pigeon under a concurrent DRL (differential reinforcement of low rate) schedule. Interresponse times falling within the two hatched areas were reinforced. The smooth curves are normal distributions fitted to the data and the area under each curve has been adjusted so that it equals the proportion of total responding in the two distributions.

of the responding is contained in the short distribution. This is in contrast to the reinforcement frequencies, since only 38% of the reinforcers were delivered for properly spaced short interresponse times. Data of this sort have been interpreted as indicating a bias in favor of short DRL values, but in the context of expectancy theory, bias is not as great as previously supposed. Subjects are regarded as choosing not between two DRL values but between two expectancies of reinforcement, and expectancy increases with decreasing time to reinforcement. Thus, some preference for a short over a long DRL may be expected on the basis of time alone.

Expectancy ratio. The theory assumes that subjects form expectancies of reinforcement associated with attempting the short or "standard" (S) and the long or "comparison" (C) DRL values, and base their choices on a ratio comparison of these expectancies. In concurrent schedules of this sort ("pacing schedules"), reinforcement for correctly spaced responding is generally arranged by a variable-interval schedule. The salient property of such schedules is that the number of reinforcers delivered over a session remains approximately constant regardless of the number of responses which meet the DRL criterion. Subjects come to respond appropriately to the rate of reinforcers, and I will assume that probability of reinforcement is assessed in terms of this rate, rather than in terms of the number of successful or unsuccessful attempts to space responding within the DRL requirement. Accordingly, reinforcement probability, p_J^*, is proportional to reinforcement rate per session, for alternative, J, $J = S, C$. The amount of reinforcement may be scaled in terms of units of expectancy, H, assuming that a standard amount of access time, A, to the reinforcer generates a total expectancy of H. The amount of expectancy generated by alternative J is then $H(A_J/A)p_J^*$, where A_J is the time of access to the reinforcer for alternative J. The DRL time window is represented by a single point (taken as the geometric mean of the window), and so expectancy of reinforcement for alternative J is given by

$$h_{J}(0) = \frac{H(A_J/A)p_J^*}{x_J}, \quad J = S, C. \quad (18)$$

This is just the original version of expectancy (Equation 1) with the motivational parameter weighted by the probability and amount of reinforcement. The time at which expectancy is

To reconcile this with the theory, one must assume an additive source of variance unrelated to the time values—for example, the suggestion in the preceding DRL analysis that motor time contributes variance to each response. For this example, motor time variance would be $\sigma_M^2 = .40$.

assessed is $t = 0$, since choices are assumed to be made at the end of each response for an attempt to time one of the two DRL values with the subsequent response. We may therefore omit the argument $t = 0$, and the expectancy ratio is

$$r_{S,C} = \left(\frac{A_S p_S^*}{A_C p_C^*}\right)\frac{x_C}{x_S}. \tag{19}$$

Choice behavior reflects this weighted expectancy ratio with choice for the short DRL occurring whenever $r_{S,C} > b$. Transforming to the unit timer, by $u = x_C/C$, $v = x_S/S$, the expectancy ratio discrimination rule becomes

$$\frac{u}{v} > b\beta S/C, \tag{20}$$

where β is the generalized expectancy ratio criterion, $\beta = (A_C p_C^*)/(A_S p_S^*)$, and u, v are independent unit timer variates. Choice probability is therefore given by the tail area of the distribution of the ratio of two samples from the unit timer. Letting $w = u/v$,

$$P(\text{"S"}) = P(w > b\beta S/C). \tag{21}$$

The relation Equation 21 is properly construed as a Weber's law for choice. It means that choice functions for constant $b\beta$ should superimpose when plotted as a function of the time ratio, S/C. It thus ties choice to time discrimination in a very direct way. One may think of schedule preference paradigms as analogous to 2-alternative, forced-choice psychophysical procedures in which the motivational parameter is weighted by the time delays being discriminated.

Weber's law for choice (Equation 21) holds independently of the form of the unit timer. However, it is clear that an efficient time estimation process should generate a steeper choice function—more extreme choice probability at a given S/C ratio—than an inefficient unit timer with a large γ.

An explicit solution for the expectancy theory choice function requires an explicit form for the distribution of the u, v. In the analysis which follows, the unit timer is assumed to be approximately Gaussian in form.[11] The distribution of the ratio of two normal variates has received surprisingly little attention in the mathematical statistics literature, considering the importance of large sample approximations in probability theory. However, Geary (1930; see also Kendall & Stuart, 1963, Vol. 1, p. 270) studied this problem and obtained an approximate solution for the distribution of $w = u/v$ when v is unlikely to assume negative values. In the context of the present account, this means that the approximation improves as the coefficient of variation becomes small. Put another way, time estimation values are by their nature positive while the normal approximation is left-truncated at zero, and it is this truncation error that introduces error into the approximate solution.

In Appendix D, an alternative derivation of Geary's solution is provided which shows that choice probability for the short delay (Equation 21) is given by

$$P(\text{"S"}) = 1 - \Phi[z(b\beta S/C)], \tag{22}$$

where Φ is the unit normal distribution function and $z(w)$ is the Geary z variate,

$$z(w) = \frac{w - 1}{\gamma\sqrt{1 + w^2}}. \tag{23}$$

The error in the approximation is obtainable from the limits of Equation 22 as the expectancy criterion, $b\beta S/C$, approaches zero and infinity. From Equation 23 $z(w) \to -1/\gamma$ as $w \to 0$, and dividing numerator and denominator by w, $z(w) \to 1/\gamma$ as $w \to \infty$. This means that choice probabilities do not reach 1 and 0 at the extremes, but rather

$$P(\text{"S"}) \to \begin{cases} 1 - \Phi(-1/\gamma), & b\beta S/C \to 0 \\ 1 - \Phi(1/\gamma), & b\beta S/C \to \infty. \end{cases} \tag{24}$$

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[11] An alternative assumption might use a Poisson process (Figure 3) for the unit timer, following Treisman's (1963) suggestion. The Poisson process results in a central F distribution for the w variate, with $2N$, $2N$ degrees of freedom where N is the number of counts required for the estimate of unity (Luce & Green, 1972). While that choice is preferable to the Gaussian assumption because of simplicity, the distribution function does not fit the data well. The reason is that the data contain sufficient variance that the chi-square variates in the numerator and denominator of F must have a low number of degrees of freedom. This parameter, $2N$, must be of the order of 1 for an approximation to the data, and this is clearly unrealistic for the form of the time estimation distributions. Timing distributions (e.g., Figure 15) are approximately normal.

Choice for S gets large as S/C gets small or as relative reinforcement, β, gets small; and choice probability gets small as these variables grow large. But the maximum and minimum values (Equation 24) are limited by the coefficient of variation. If γ is sufficiently small, choice probabilities approximate 1 and 0 at the extremes. However, for intermediate γ, these limits are just the truncation error introduced by assuming positive values of the time estimates.

The ratio, w, is "Cauchy"-like. It is distributed according to the Cauchy density when the numerator and denominator are unit normal variates (Feller, 1966, pp. 50–51). For the unit timer variates in the present application, this condition is approached as γ grows large, since dividing u, v by σ results in a ratio of normal variates with standard deviation 1 and mean $1/\gamma$. Of course, this is also just the condition for which Geary's approximation is not viable, as the truncation errors (Equation 24) increase with γ. In the limit the choice function is flat at indifference, since the truncation errors converge on $\frac{1}{2}$.

The Cauchy distribution has the interesting property that it is an exception to the central limit theorem. Averages of samples of w are distributed as w with no improvement in variance. In the present context, this means that the expectancy ratio is particularly inefficient as a decision statistic since a strategy such as averaging, which generally improves accuracy, is not available. Moreover, it is just when γ is relatively large, and w therefore relatively Cauchy-like, that such a strategy might be most appropriate.

The Cauchy density looks like the normal density but approaches the x-axis very slowly, so that the integrals for the mean and variance do not converge. In the present context, this means that choice functions reach the upper and lower limits very slowly in the variate $b\beta S/C$, even for subjects with relatively high sensitivity to time. In later applications we will see that the range for the choice functions generally exceeds two orders of magnitude.

The expectancy theory choice function (Equation 22) will be applied to relevant data below, but first an empirically based alternative choice formulation is developed with which to contrast the present account.

Matching. Following Herrnstein's influential work (1961, 1970), much empirical and theoretical effort has been devoted to the analysis of choice in concurrent schedules of reinforcement when the alternatives are variable-interval schedules. While the present account does not bear directly on these efforts, several workers have modified the original "matching law" for situations in which the alternatives are scheduled with fixed time values as in concurrent DRL. The formulation originally proposed by Chung and Herrnstein (1967) asserted that choice probability should match the relative "immediacy" of reinforcement, where immediacy is defined as the inverse of the time to reinforcement. This is, of course, quite close to the expectancy idea, and one might even regard expectancy as a stochastic version of relative immediacy with a motivational and a discriminability parameter. Several workers have modified this account in different ways (e.g., Baum & Rachlin, 1969; Catania, 1963; Fantino, 1969a, 1969b; Killeen, 1968; Shimp, 1969), and a comprehensive survey will not be attempted here. Rather, a simple generalized version of the matching law will be proposed which captures the thrust of many of the modifications. The development follows that by Baum and Rachlin (1969) who argue that the "value" of a response alternative is the ratio of the amount to the delay of reinforcement for that alternative. The matching law on this formulation asserts that the ratio of responses should equal the ratio of the value of the alternatives, $(A_S/S)/(A_C/C)$. Generalizing the value ratio to include probability of reinforcement and bias, choice probability for S is then

$$P(\text{"S"}) = \frac{A_S p_S^*/S}{A_S p_S^*/S + bA_C p_C^*/C} = \frac{1}{1 + b\beta S/C}. \tag{25}$$

Equation 25 differs from some constructions of matching, but the differences generally appear when the time values, S, C are not fixed (e.g., Killeen, 1968; Fantino, 1969a, 1969b). Other variants have also been proposed for the concurrent-chains paradigm (e.g., Davison & Temple, 1973; Squires & Fantino, 1971) and that development will be treated later.

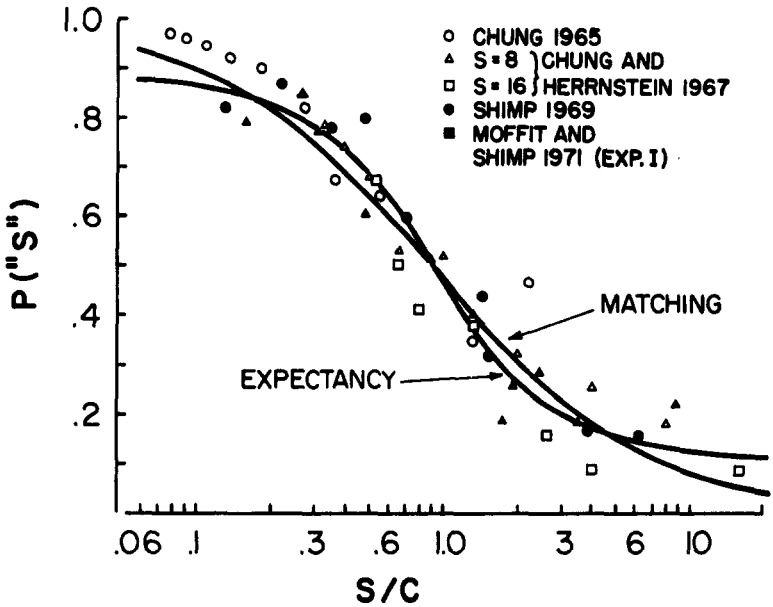

Figure 16. Preference for the standard delay or shorter DRL (differential reinforcement of low rate) value, P("S"), as a function of the log of the ratio of the standard (S) to comparison (C) delay or DRL, from four experiments. Open points are data from studies varying delay and filled points from studies varying DRL requirement. The point values used were the geometric means of the DRL windows. The theoretical functions are best fits for expectancy theory and matching.

However, most of the matching laws converge to Equation 25 for choice between fixed-time alternatives with varying amounts and probabilities of reinforcement.

The present account and the matching law are alike in specifying that choice probability ought to be constant with constant values of the ratio of time to reinforcement (Equations 21 and 25). Thus, they both represent a generalized Weber's law for choice. They differ in that the present account does not require response probability to match relative immediacy. Variation in the efficiency of the unit timer may result in (a) near convergence to the matching law, or (b) divergence, either in the direction of "undermatching" in which choices are closer to indifference than predicted by Equation 25, or in the direction of "overmatching" in which choices are more extreme (Baum, 1974).

Delay of reinforcement. One method of varying the estimated time to reinforcement is that discussed above in which different DRL values are concurrently reinforced. An alternative method devised by Chung (1965) and Chung and Herrnstein (1967) varies the delay of reinforcement directly without requiring a well-timed response to produce it. In their experiments, pigeons were exposed to a concurrent procedure in which reinforcement was programmed on equal variable-interval schedules on two concurrently available response keys. However, responses on one alternative produced a blackout for a standard delay (S) ending with reinforcement, while responses on the other produced a comparison delay of reinforcement (C). Subjects were studied at a variety of standard and comparison delays, which were not changed until responding had stabilized. The comparable procedure in concurrent DRL has been studied by Shimp (1969) and by Moffit and Shimp (1971, Experiment I). In the Shimp study, two DRL values were studied with one response key which changed color during the DRL reinforcement availability periods. In the Moffit and Shimp study, the two DRL values were programmed for reinforcement on two concurrently available keys. In this work and the delayed reinforcement studies, reinforcement probability (which is proportional to the VI value in our construction of expectancy, Equation 18) was equal for the two response alternatives so that $\beta = 1$. Data from the four studies are shown

in Figure 16.[12] The probability of a choice of the standard (or the shorter DRL) is plotted as a function of the log of the time ratio, S/C. When this ratio is small, choices for the standard predominate, and this probability decreases systematically as the standard becomes large relative to the comparison. The two solid functions represent the matching relation (Equation 25) and the expectancy theory formulation (Equation 22) fitted to the data by a modified staircase method.[13] The matching function is shallower and deviates from the data in the middle range of time values in the direction of underestimating choices for S (C) when $S < C$ ($S > C$). The expectancy theory formulation fits the data in the middle range of the time values somewhat better but deviates from the data at the extremes, since choice probability in the approximation is constrained by the truncation errors (Equation 24). The data were fit using a staircase method and the fits resulted in virtually no bias ($b = 1.1$). The bias value may be read from the figure as the indifference point, where the functions intersect at an x-axis value of $1/b$. The sensitivity index ($\gamma = .8$) in the expectancy formulation is comparable to that for asymptotic time discrimination data.

The expectancy theory choice function results in a somewhat better fit to the data over the intermediate range of choice probability and a somewhat worse fit to the data at the extremes. The variance accounted for by both functions is virtually identical, $\omega^2 = .936$ for the expectancy function and $\omega^2 = .935$ for the matching function. Thus, there is little to choose between the two approaches when all the data (including the extremes) are considered. However, the deviation from matching in the midrange of choice probabilities is worth noting, since it is a deviation which is not predicted by any of the matching law variants, and it is one that will reappear in the concurrent-chain context in which more precision in timing is found.

The failure to find a significant bias toward short delays or short DRL values is important in the light of a common report in favor of short times in other contexts. If anything, the present data slightly favor long choices ($b > 1.0$).

Amount and probability of reinforcement. Several authors have studied amount and probability of reinforcement in concurrent DRL procedures. Staddon (1968) studied the frequency of reinforcement variable (e.g., Figure 15) with one key, and Shimp has studied both frequency and amount of reinforcement in a similar context (Shimp, 1968). A later study by Moffit and Shimp (1971) contained one experiment (Experiment II) in which two groups of birds were studied on DRL values which corresponded closely to Staddon and Shimp's earlier procedures (Groups A and B, respectively) but the DRLs were associated with two keys. The experimental variables were again amount and frequency of reinforcement. In this work, the standard (S = short) and comparison (C = long) DRL values were maintained constant, and relative access and probability of reinforcement were varied.

Preference for the short or standard DRL is plotted as a function of $\beta S/C$ on a log axis in Figure 17. The one key data (open points) and two key data (filled points) were taken from the studies listed in the legend. There is considerably more scatter in these data than those for varying time alone, but it appears unrelated to whether choice is studied in the one key or two key situation. The smooth curves are the fits for the expectancy theory choice function and for the matching relation.

Two features of these data are important. First, there is a relatively strong bias in favor of preference for the short DRL. The bias may be read off of the preference functions as the inverse of the abscissa value at which indiffer-

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[12] Data from the experiment by Chung (1965) were adjusted by allowing "immediate reinforcement," $S = 0$, to take on a value estimated by Chung and Herrnstein (1967) as between 1 and 2 sec, for the time required for birds to pick up the "immediate" reinforcer. The value used was 1.3 sec, and this time was subtracted from the access time.

[13] The procedure for fitting these data and others to be presented later consisted in choosing reasonable values of b and γ by inspection and then holding b constant and varying γ until the variance accounted for (ω^2) reached a maximum. Then b was set at a new value and γ varied until ω^2 again reached a maximum. If the second maximum ω^2 was less than (greater than) the first, b was changed in the opposite (same) direction and a new range of γ examined. Iteration was stopped when changes in the variance accounted for appeared small (of the order of 1 or 2%).

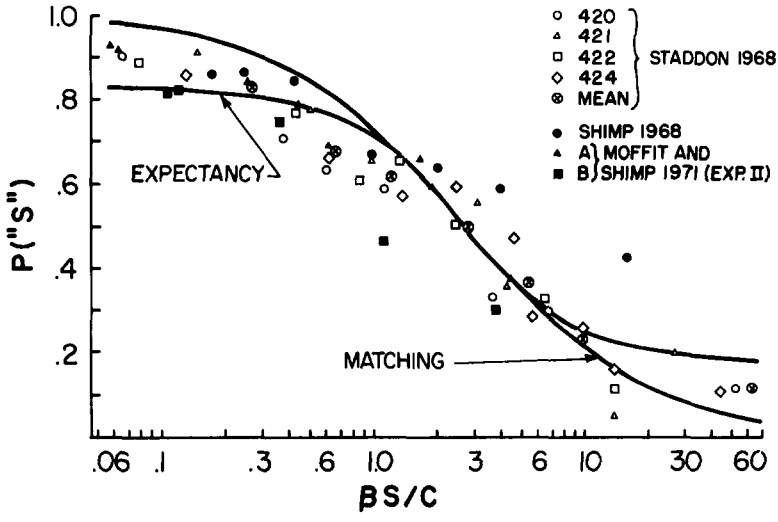

Figure 17. Preference for the standard or shorter DRL (differential reinforcement of low rate), P("S"), value as a function of $\log \beta S/C$ from three experiments. The standard and comparison delay values (S,C) were held constant, and reinforcement differential, β, was manipulated by changing the variable-interval schedules or access times associated with either alternative. The functions are the best fits for expectancy and matching.

ence is obtained ($b = .375$). This means that reinforcement amount or probability must favor the long DRL value by about a 5 to 2 margin for birds to be indifferent to the two alternatives. This finding is to be contrasted with the failure to find a significant bias when time alone is the variable manipulated. It strongly suggests that the observed preference here is actually a preference for the smaller amount or lower probability of reinforcement associated with the short DRL. If the true bias value were $b = 1$, then β at indifference would be less than 1. That is, short access times may result in more relative value than long access times. This suggests that the amount by which expectancy is increased by increasing access to the reinforcer is an exponential growth function of the access time, rather than, as proposed here, a strictly proportional function. This is clearly a more reasonable assumption when one considers the extremes of access to the reinforcer. It would be unlikely that the increment in reinforcer duration from, say, 1 sec to 2 sec would be equalled by the increment from, say, 50 sec to 100 sec. It is more plausible that an increment in reinforcement access at low levels of the value of the reinforcer represents a proportionally larger increment than at high levels.

While bias for the short DRL is substantial in these data, it is nothing like the bias estimated without taking into account the time to reinforcement. To obtain indifference between the two alternatives here disregarding the time to reinforcement, the values required would be of the order of 10 to 20—that is, about two to five times the indifference values shown here. The matching formulation appropriately corrects for time to reinforcement (Shimp, 1969) as does the present formulation, thus reducing the magnitude of the obtained bias considerably.

The second point to note is that the choice data are shallower here when amount and probability of reinforcement are varied than when time alone is varied (Figure 16). This is reflected in a larger coefficient of variation ($\gamma = 1.05$) in the expectancy theory formulation. One of the results of a larger γ is a greater deviation from the data at the extremes because of the truncation error (Equation 24). Nevertheless, the variance accounted for by the fit is .860, and this is to be contrasted with .781 for the matching function. The flatter slope here may reflect additional sources of

variance in the choice mechanism associated with unequal rates and amounts of reinforcement (e.g., the context effects for reinforcement amount and probability discussed earlier). In the Shimp experiment, reinforcement was probabilistic, but subjects were given an exteroceptive cue on the key during the time window for each DRL. In the other two studies, (probabilistic) reinforcement was delivered after some but not all of the interresponse times which met the DRL time window requirement. Thus, in these studies, subjects were unable to discriminate appropriately spaced responding which was unreinforced from interresponse times which were either too short or too long to meet the DRL requirements and went unreinforced for that reason. This ambiguity may have resulted in additional variance in the time estimation process assumed to underlie choice behavior.

A more fundamental caveat is indicated by the results of an experiment by Hawkes and Shimp (1974). They studied concurrent DRL in a one-key situation with constant reinforcement rate and varied the absolute size of S and C (the DRL values), keeping their ratio approximately constant. They found that preference for S increased with increasing absolute value of the S, C pair, contrary to either formulation of Weber's law for choice. Their result is similar to a finding in concurrent-chain schedules, namely, that there is a more extreme preference for the shorter of two terminal links when both terminal links are long than when they are both short.

The correspondence suggests that the DRL timing intervals may be functionally similar to the terminal links in concurrent chains; and the VI pacing schedule arranging reinforcement opportunities, particularly when interreinforcement intervals are relatively long as in the Hawkes and Shimp experiment, may be functionally similar to the initial link in concurrent chains. This is in contrast to the present interpretation of the VI schedule in terms of reinforcement probability. In the application of expectancy theory to the concurrent-chain situation we will see that it is just when initial link times are relatively long that an effect of the absolute values of S and C is observed.

A similar interpretation may be made of the concurrent delay of reinforcement studies (Chung, 1965; Chung & Herrnstein, 1967). There the delay intervals, which were associated with blackout stimuli, may be identified with terminal links, and the VI schedules with an initial link. These interpretations suggest that there may be effects attributable to the absolute size of S, C values in concurrent schedules which contribute additional variance to the data in Figures 16 and 17.

In general, additional sources of variance in choice paradigms result in a flattening of the choice functions, which might be termed "regression toward indifference." Unfortunately, it is all too easy to name several other potential sources of such variability. For example, variance in the bias parameter might contribute, as well as variation in "attention" from trial to trial. Occasional inattention would result in choice proportions which were a weighted average of (a) indifference, when subjects were not attending at all, and (b) the true choice proportions. It is important in this context, then, that efficient choice behavior may be demonstrated under appropriate conditions. For the concurrent procedure, temporal manipulations result in sharper discrimination performance than reinforcement manipulations, and this sharpening of the slope of the choice function is increased still further under the concurrent-chain paradigm treated next.

Concurrent-Chain Schedules

Since its introduction by Autor (1960), the concurrent-chains procedure has been widely used to study preference for a variety of complex reinforcement schedule outcomes. In the typical procedure, birds respond on either of two concurrently available keys illuminated with the same color during an "initial link." Occasionally, responses on either key result in a change in the color on that key and a blackout on the other key. With this change to a "terminal link," the schedule is also changed and responses are effective only on the illuminated key. The duration of the initial link during which responding may occur on either key is generally variable with the same mean for the two responses, and the schedule of reinforcement on the two keys during the

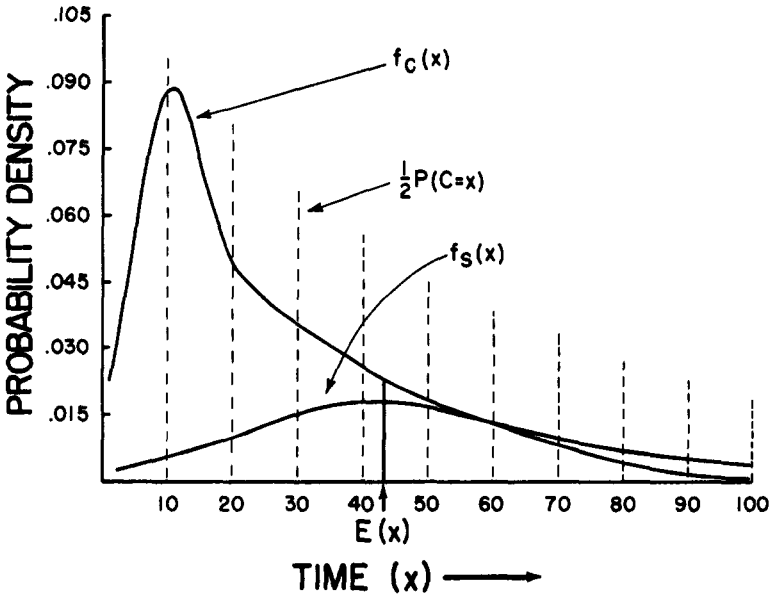

Figure 18. Estimation distributions assumed to underlie choice between fixed versus variable delays. The functions have the same mean (43), and the bell-shaped curve $f_S(x)$, for the standard, S, is a normal distribution centered on this value. The skewed distribution $f_C(x)$, for the comparison, C, was obtained using the scalar property for the mixture of different point delays indicated by the dashed discrete distribution $P(C = x)$. The skew in the comparison distribution strongly favors short estimates.

mutually exclusive terminal links is the experimental parameter of interest. For example, Autor (1960) and Herrnstein (1964b) showed that the matching relation approximately described the proportion of initial link choice responses when the two terminal links were associated with different VI schedules of reinforcement. Both Autor (1960) and Herrnstein (1964a) also showed that the matching relation was *not* an appropriate description for choice responding when one of the terminal links was a fixed-interval schedule and the other a variable-interval schedule. Rather, the variable terminal link was consistently preferred when its mean was equal to the FI value. A variety of workers have since confirmed and extended this finding of a decided preference for aperiodic over periodic reinforcement schedules (Cicerone, 1976; Davison, 1969; Fantino, 1967, Killeen; 1968; Sherman & Thomas, 1968). The finding is consonant also with earlier reports of a preference for variable versus fixed delay of reinforcement with rats in a runway situation (Logan, 1965; Pubols, 1962). Several accounts have been proposed as quantitative assessments of this deviation from matching. Killeen (1968) suggested the harmonic mean of the VI as the appropriate measure of reinforcement rate. Fantino (1967) found the geometric mean to be best, while Davison (1969) suggested the cube of the harmonic mean as the appropriate transformation. A later version of Fantino's approach (Fantino, 1969a, 1969b; Squires & Fantino, 1971) takes into account effects attributable to long versus short initial link schedules as well. However, no satisfactory single account appears to encompass all the data.

The analysis proposed here does not settle this matter, but it does suggest reasons for why different averaging procedures appear to work better or worse in different laboratories.

Consider the two distributions for terminal link reinforcement times shown in Figure 18. The curves represent time estimation distributions for a point estimate of 43 sec (Gaussian distribution) and for a geometric mixture of time values with a mean of 43 sec (skewed distribution). Both curves were drawn assuming the scalar property and a normal distribution of estimates of unity with $\mu = 1$ and $\gamma = \sigma = .5$. The mixture was generated by

an approximately geometric distribution on multiples of 10 sec with $p \simeq .2$. The mixing distribution (with ordinate values rescaled by $\frac{1}{2}$) is indicated by the dashed lines. Suppose that the standard is the fixed interval, and subjects obtain an estimate during the initial link of a concurrent-chain schedule of x_S and x_C, where x_C is drawn from the mixture. If responding during the initial link reflects the frequency with which the ratio of expectancies based on these estimates exceeds b, a very sizeable preference for the variable alternative results. The reason is that the scalar assumption forces a high concentration of density at the low values of the mixture. The extent of the deviation from .5 in choice for the variable alternative depends both on variance in the mixing distribution and on variance in the subject's unit timer. Increases in either decrease, but do not eliminate, the preference for the variable alternative. The manner of constructing the variable schedule varies somewhat from laboratory to laboratory, and this diversity undoubtedly contributes to diversity in the quantitative proposals. In the present context, this difficulty precludes an exact treatment of the choice between fixed and variable delays, since the mixing distribution has a profound effect on the degree of skew in the resulting estimation distribution. Moreover, mixtures of this sort are theoretically intractable assuming a Gaussian unit timer. The treatment which follows will focus on choices for different delays and amounts of reinforcement in the terminal links, when these parameters are fixed.

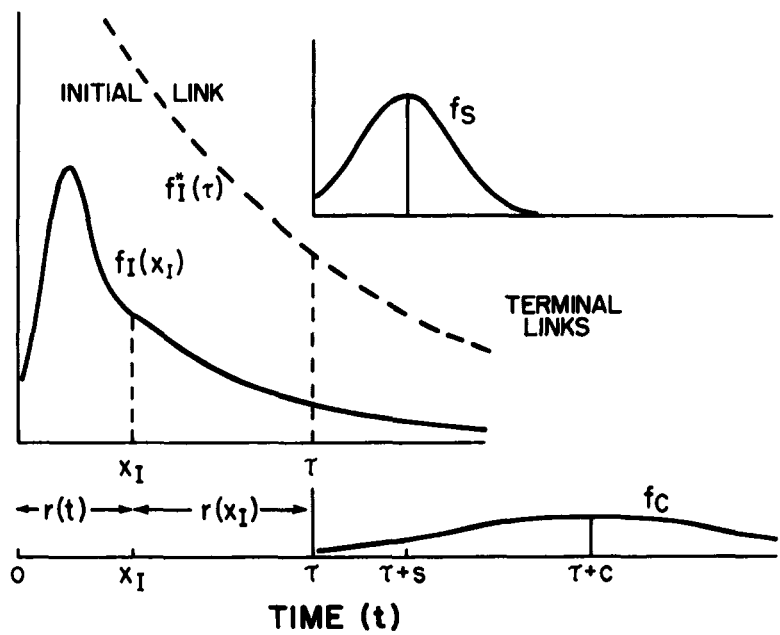

Figure 19. Schematic representation of time estimation for the concurrent-chains paradigm. At time $t=0$, (bottom axis) the initial link (I) begins and the concurrent response keys remain available for, in this example, τ sec. At this point, one of the two terminal links begins. The terminal link ends with reinforcement after either the standard, S, or comparison, C, delays. The solid distribution curves are those associated with estimates of initial link time, f_I, or terminal link times, f_S, f_C. The terminal link distributions are normal with the scalar property and the initial link distribution is the mixture formed from scalar estimates of initial link times distributed as f_I^* indicated by the exponential dashed line function. A sample estimate of initial link time is shown by the dotted line at x_I. Between the onset of the initial link, and the point at which $t = x_I$, the expectancy ratio is a joint function of the terminal link estimates, the initial link estimate, and t. Between the time when the initial link is expected to terminate, at x_I, and at the time when it actually terminates according to the variable-interval schedule (here at τ), the expectancy ratio is a function of expected time to reinforcement in the terminal links only.

Expectancy ratio. A schematic representation of the concurrent-chain situation is shown in Figure 19. Along the bottom a time axis for a cycle of an initial link followed by either terminal link is shown from left to right. During the initial link, there is a distribution of initial times, τ, with a mean, $E(\tau) = I$, programmed by the experimenter. An exponential form has been assumed here and is represented by the dashed line. Beginning at time, $t = 0$, the initial link keys are both illuminated and choice responding continues until $t = \tau$, when a response on one or the other key produces the associated terminal link. The distributions shown for the terminal links are the estimates of S and C, as in our preceding applications, with the scalar assumption. The additional feature required in the concurrent-chains procedure is an estimate of the initial link time, x_I, and these estimates are distributed as the mixture, $f_I(x_I)$. At the beginning of the initial link three time values are estimated x_I, x_S, and x_C. Expectancy of reinforcement associated with a choice of either response alternative is then formed in a manner analogous to that for concurrent schedules, except that the estimate of time to reinforcement is now the sum of the estimate of terminal link time and the estimate of initial link time. As time in the initial link (t) elapses, the time to reinforcement is decreased by t, analogously to the expectancy functions for points within a fixed-interval cycle in the first application of the theory. Expectancy at time, t, within the initial link is therefore

$$h_J(t) = \begin{cases} \dfrac{H(A_J/A)p_J^*}{x_J + x_I - t}, & t < x_I < \tau, \text{ or } t < \tau < x_I \quad (26a) \\ \dfrac{H(A_J/A)p_J^*}{x_J}, & x_I < t < \tau, \quad J = S, C. \quad (26b) \end{cases}$$

When x_I overestimates the initial link time actually programmed, $\tau < x_I$, Equation 26a is appropriate, since choice responding terminates at $t = \tau$. In Figure 19, a sample x_I is indicated which underestimates τ, and here Equation 26a is appropriate only for t values between 0 and x_I. When the elapsed time in the initial link equals or exceeds the estimated time for the duration of that link, subjects are viewed as expecting onset of a terminal link immediately. Thus, for $x_I < t < \tau$, expectancy in the initial link is equivalent to expectancy in the concurrent situation, and Equation 26b holds.

As before, choice for the standard, S, occurs when the expectancy ratio exceeds the bias value, b. From Equation 26, this ratio is

$$r_{S,C}(t) = \begin{cases} \left(\frac{1}{\beta}\right)\dfrac{x_C + x_I - t}{x_S + x_I - t}, & t < x_I < \tau, \text{ or } t < \tau < x_I \quad (27a) \\ \left(\frac{1}{\beta}\right)\dfrac{x_C}{x_S}, & x_I < t < \tau, \quad (27b) \end{cases}$$

where β is the motivational parameter, $\beta = (A_C p_C^*)/(A_S p_S^*)$. The expectancy ratio intervals appropriate to x_I, τ, are indicated in Figure 19. For short estimates of initial link time, most of the choice responding is exactly comparable to choice under the concurrent procedure, where the ratio of time to reinforcement in the terminal links weighted by the motivational differential is the decision statistic. For long estimates of initial link time, the expectancy ratio changes as a function of how much time has elapsed during the initial link. However, under certain circumstances, initial link times may be disregarded, and this situation is treated first.

Delay of reinforcement. Perhaps the clearest demonstration of the precision of the concurrent-chains technique and the clarity of the results is provided by an experiment by Killeen (1970). He studied pigeons in a concurrent-chains procedure in which the initial link was a variable interval, 60-sec schedule and the terminal links were two unequal fixed-interval schedules. One key (and color) was associated with a standard terminal link FI schedule ($S = 20$ sec), and the comparison FI in the other terminal link was set at $C = 5$, 14, 30, or 60 sec, and remained at one of these values until stability in choice responding had been achieved. Reinforcement was the same for both terminal link alternatives, and the data were remarkably free of bias ($b\beta \simeq 1$).

For this situation, the expectancy ratio criterion is exceeded when $r_{S,C}(t) > 1$. But from Equation 27, this is equivalent to $(x_C/x_S) > 1$, independently of the value of the initial link estimate, x_I. This is because the

numerator and denominator of the expectancy ratio are both increased by the same amount $(x_I - t)$ at any point in the initial link. Translating to the unit timer variates, $u = x_C/C$, $v = x_S/S$, the decision criterion reverts to Equation 20, $(u/v) > (S/C)$, the criterion rule for the concurrent paradigm with $b\beta = 1$. The simplicity of this result unfortunately requires absolutely no bias in individual birds, since introduction of bias allows variance in x_I to inflate numerator and denominator variance in the expectancy ratio (Equation 27a). However, in Killeen's experiment, this condition is very nearly approached, with the important result that birds show overmatching of choice responding relative to the matching relation (Equation 25).

The data are presented in Figure 20 with probability of choice for the standard on the ordinate and the ratio S/C on a log scale on the abscissa. Points are for individual birds and the circled Xs represent means. Two functions are shown: the expectancy function (Equation 22) and the matching function with the same bias value.[14] The data for all subjects deviated systematically from the matching function, as Killeen noted, in the direction of a more pronounced choice for the shorter of the two FI alternatives. The theory was fit to the data with a staircase method as in previous applications, and the resulting function has $\gamma = .45$ and virtually no bias ($b = .95$). The expectancy theory function represents a good fit ($\omega^2 = .953$), and this is in contrast to the systematic deviation from the matching function ($\omega^2 = .786$).

Two points are important here. First, the formulation is identical to that for concurrent choice between fixed point alternatives, and the earlier application was also found to be unbiased (Figure 16). This means that when there is no reinforcement differential and the variable under study is time alone, birds show no preference for short versus long time to reinforcement when these are computed in the expectancy ratio manner. Second, only a small deviation from matching in the direction of more extreme choices for the preferred alternative was found for the concurrent paradigm (overmatching in Figure 16). Overmatching is substantially amplified here. The finding corresponds in the theory to efficient time estimation, and the sensitivity value ($\gamma = .45$) corresponds very well to those for FI breakpoint data ($\gamma \simeq .35$, Figure 5).

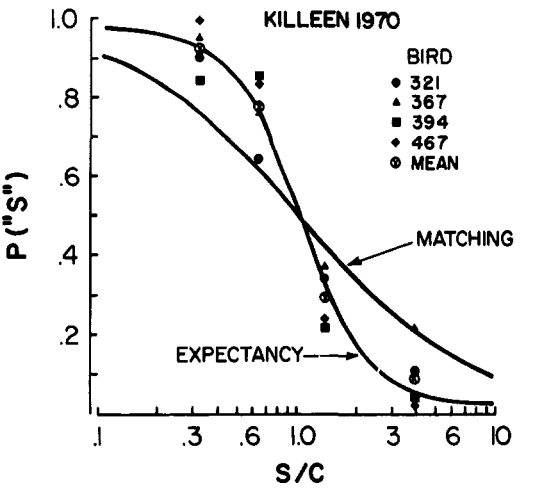

Figure 20. Preference for the standard, P("S"), as a function of the log of the ratio of standard to comparison, S/C, for four pigeons under the concurrent-chains procedure with the best fit expectancy and matching functions. Note that preference exceeds matching in the direction of more extreme preference for whichever is the shorter alternative (overmatching).

The theoretical account of overmatching is not vulnerable to a criticism that may be leveled against accounts of deviations below matching. If the experiment is conducted in such a way that uncontrolled sources of variance enter into choice responding, then one expects regression toward indifference, rather than an increase in preference from the matching prediction. These data therefore testify to the precision of the concurrent-chain technique in assessing sharp time discrimination.

None of the proposals for variants of the matching law predict overmatching. Chung and Herrnstein's (1967) relative immediacy transformation is equivalent to Fantino's (1969), Killeen's (1968), and Shimp's (1969) proposal of matching to relative harmonic means. Later variants of this idea for the concurrent-chain paradigm (Davison, 1976; Davison & Temple, 1973; Fantino, 1969a, 1969b; Navarick & Fantino, 1976; Squires &

[14] Bias values were permitted to vary in the fitting procedure to test the assumption that $b \simeq 1$.

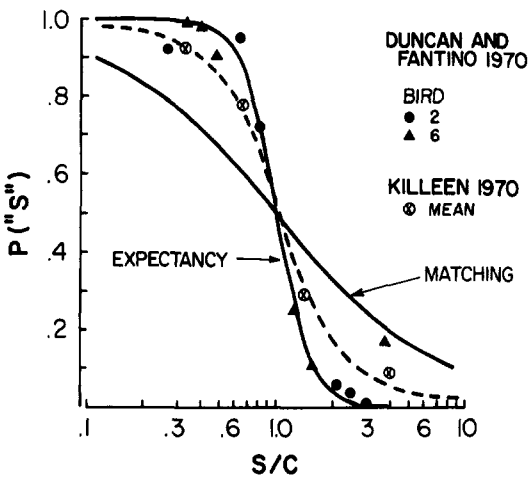

Figure 21. Choice for the standard, P("S"), as a function of the log of the ratio of standard to comparison, S/C, for two pigeons in the concurrent-chains paradigm. The dashed curve is the expectancy function for the Killeen data (circled Xs) shown in Figure 23. Again, note the overmatching.

Fantino, 1971) predict a reduction in choice proportions toward indifference with the introduction of relatively long initial link times. These variants of the matching relation are all introduced to account for undermatching, or regression toward indifference. Approximations to matching or undermatching in the present theory result from decreased sensitivity to time. As γ grows large, the expectancy theory choice functions flatten toward indifference.

The clear implication of this line of argument is that the finding of approximate matching may result from the introduction of additional variance in the estimates of the time values being compared. Such variance is introduced deliberately by the use of VI schedules as the choice alternatives, and this of course is where the matching law originated. However, added variance may also arise inadvertently from several sources. For example, if subjects are not studied for a sufficiently long period of time, or if subjects with different response biases are averaged, or if other sources of variance such as "inattention" are introduced, choice will regress toward indifference.

Since introduction of any uncontrolled variance in the experimental paradigm tends to flatten the psychometric choice functions, it is important to demonstrate that substantial overmatching may be obtained in different laboratories. Duncan and Fantino (1970) studied pigeons in a concurrent-chains procedure in which terminal links were reinforced either according to fixed-ratio schedules or (for two subjects) fixed-interval schedules. The results for the subjects with FI terminal links were quite comparable to those of Killeen, and their data are shown along with the mean values from Killeen's study in Figure 21. The two solid curves in the figure represent the matching function and the best fit expectancy function for the Duncan and Fantino data. The dashed function is the fit for Killeen's data (Figure 20). The fit to the Duncan and Fantino data is very close ($\omega^2 = .977$) and much better than that for the matching function ($\omega^2 = .680$). The sensitivity parameter value here ($\gamma = .25$) is the lowest for any of the data treated previously so that the overmatching result is dramatically confirmed. The best fitting bias value was $b = 1.0$ ($\pm .05$), and thus again, when time to reinforcement is the variable, there is no bias in favor of the shorter time. These two studies speak to the precision of the concurrent-chains procedure in revealing precise control by time, with γ values comparable to those from the FI breakpoint data.

Amount versus delay of reinforcement: self-control. Following Rachlin's original insight (Rachlin, 1970, 1974; Rachlin & Green, 1972), a recent literature has emerged studying the relative contribution of delay and amount of reinforcement when these two variables are pitted against each other. The typical investigation has studied choice for a small reward at a short delay versus a larger reward at a longer delay. The important finding is that when both delays are relatively short, subjects choose the small reward with the small delay—thus exhibiting "impulsiveness" (Ainslie, 1974, 1975). But, when both rewards are further away in time, subjects may choose the larger reward at the longer delay—thus exhibiting "self-control" (Rachlin, 1974).

The choice for the larger reward when both rewards are further away in time has been proposed as an animal analogue of "commitment" to the more rational alternative. Aside from the obvious implications for a behavioristic view of impulse control in humans, the

finding represents an interesting variant of the choice formulation since it implies a reversal of preference which is not immediately obvious from the expectancy theory account or from the matching law. However, as Rachlin and Green (1972) point out, the ratio of time and amount of reinforcement in the matching law predicts such a reversal if the standard and comparison times are both increased by a constant, rather than by a multiple. For example, in Equation 25, if the larger amount of reinforcement is twice the smaller ($\beta = 2$), then preference for the small reward at the shorter delay should occur when $S/C < \frac{1}{2}$ (assuming no bias), and preference for the larger ought to occur when $S/C > \frac{1}{2}$. This means that values of the time ratio of about 1.0 should produce preference for the larger reward, and this is approximated by adding a large amount of time to both the standard and comparison delays. This reversal, of course, follows from the expectancy theory formulation (Equation 22) as well.

Parametric data on the phenomenon are not extensive as yet, but Rachlin and Green's (1972) study provides an illustration. The experimental paradigm was a variant of the concurrent-chains procedure, but with an initial link in which subjects were required to emit 25 responses for the onset of the terminal links. Terminal links were fixed-interval schedules with a small amount of reinforcement at S sec or a larger amount of reinforcement at C sec. The fixed ratio requirement in the initial link meant that the time in this link was controlled by the subjects. If they emitted 25 responses relatively rapidly, the onset of the terminal links was correspondingly rapid. The authors report that preference in the initial link on a given trial was generally unilateral for one or the other terminal link key. While the time in the initial link was not invariably short, it tended to be composed largely of a pause at the onset, with a rapid emission of the required 25 responses once responding had begun. It seems reasonable, then, to suppose that behavior in the initial link did not reflect initial link time, but rather that choice was based only on estimates of reinforcement amount and delay in the terminal links. Accordingly, the expectancy theory choice formulation is equivalent to that for the concurrent paradigm, and Equation 22 should apply.

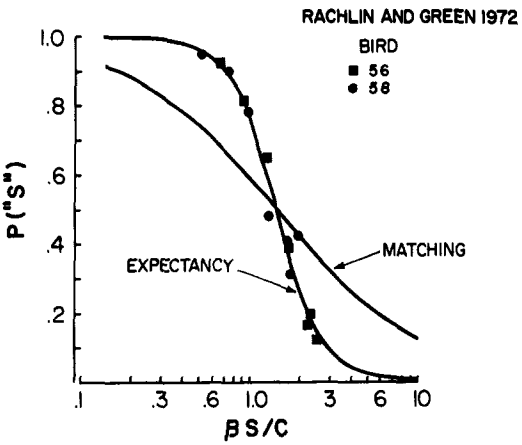

Figure 22. Preference for the shorter delay, P("S"), associated with a smaller reward as a function of the log of the ratio criterion, $\beta S/C$, for two pigeons exposed to a variant of the concurrent-chains paradigm. The standard (S) and comparison (C) delays were varied and reinforcement for the shorter was always less (2 sec access) than reinforcement for the comparison (4 sec access). "Self-control" is indicated by the (rational) choice of the longer delay and larger reward which occurs at values to the right of the indifference point. "Impulsiveness" is indicated by choices for the shorter delay to the left of this point. Bias in favor of the shorter delay and smaller reward is reflected in the shift of the indifference point to an x-axis value greater than $\beta S/C = 1$.

Four subjects were studied at a variety of delay values, and all showed some increase in preference for the larger reward at the longer delay as both S and C were increased. Two of these subjects showed graded choice responding over a range of delays spanning nearly complete choice for the shorter to nearly complete choice for the longer delay. Their data are replotted in Figure 22. Points represent choice probabilities for the shorter delay as a function of log $\beta S/C$. Each subject had slightly different bias values and their data have been shifted laterally so that indifference points coincide at the average bias value ($b = .7$), The two smooth functions are those for expectancy theory and matching. Again, substantial overmatching is obtained. Preference for either alternative is more extreme than predicted by matching. This is reflected in a large difference in variance ac-

counted for (expectancy $\omega^2 = .964$, and matching $\omega^2 = .622$). Overmatching again results from efficient time estimation, and the sensitivity parameter ($\gamma = .35$) is appropriately low. Efficient timing here is particularly striking in view of the reduced sensitivity observed previously when reinforcement amount and probability were varied in the concurrent procedure (Figure 17). As in those data, a clear bias favoring the shorter delay interval was found ($b = .7$), but in the present case, only two amounts of reinforcement (2 sec and 4 sec) were used. Possibly the variety of amounts and probabilities of reinforcement in the concurrent studies contributed to variance in the choice data there.

The combination here of efficient time estimation and substantial bias lends further support to the possibility that access time to the reinforcer may not be a direct reflection of expectancy or "value" of the reinforcer. If in the present case, access to the smaller reinforcer represented relatively greater reinforcement value than access to the larger reinforcer, then these data may reflect no true bias and are simply plotted shifted laterally by the (true) value of the ratio of expectancies for the smaller and larger reward.

The central finding here is that preference shifts toward the larger reward as the absolute amount of time preceding both rewards is lengthened. This corresponds in the theory to S/C approaching 1.0 so that the reinforcement differential, β, more nearly determines choice. Both the matching formulation and expectancy theory appropriately predict this shift, but matching badly underestimates the slope of the obtained functions.

The self-control finding is relatively new, and little parametric data are available. However, several qualitative findings of overmatching and undermatching when initial link as well as terminal link times are varied are available. The theoretical treatment of these effects requires a generalization of the Geary z variate solution for the expectancy ratio (Equation 27).

Initial link time. Navarick and Fantino (1976) have shown that in the self-control paradigm, preference for the larger reward increases with increasing absolute time in the terminal links even when these times are equal. Also, MacEwen (1972) demonstrated increasing preference for the shorter of two terminal link FIs when their absolute values were increased but their ratio held constant. The Hawkes and Shimp (1974) result discussed earlier is the analogue of this finding in concurrent DRL. As long as the ratio of S to C is held constant, both the matching relation and expectancy theory predict no change in choice. Alternative formulations (e.g., Davison, 1976; Davison & Temple, 1973; and Squires & Fantino, 1971) argue that this effect depends upon taking into account initial link time. Conversely, Wardlaw and Davison (1974) and others have shown that when initial link times are increased substantially, preference shows a regression toward indifference. Choice is less extreme for the preferred alternative as initial link time is increased. Thus, increasing initial link time tends to flatten the psychometric choice function, while increasing terminal link times tends to steepen it.

In the theory, both of these effects depend upon some degree of reinforcement differential or bias ($b\beta \neq 1$) so that variance in initial link time estimates, x_I, degrades the precision with which the discrimination between S and C may be made. In Appendix E, an approximation for the distribution of the generalized expectancy ratio is developed, which takes into account this additional variance. The approximation regards choice behavior as "typified" by choice at an intermediate point within the initial link, which is close to the estimate, x_I. At this point within the initial link, subjects expect the onset of the terminal link soon, and so their choices are largely based on estimates of time to reinforcement in the terminal links. However, variance in the initial link estimate remains in the expectancy ratio, so that discrimination between S and C is more difficult. In Appendix E, choice probability is found as

$$P(\text{"S"}) = 1 - \Phi[z'(w)], \qquad (28)$$

where $z'(w)$ is the generalized Geary z variate, and $w = b\beta S/C$. The generalized z variate is similar to the original, Equation 23, but contains a denominator term reflecting variance in initial link times, $V^2 = Var(\tau/I)$, so that $z'(w)$ approaches zero as this variance gets

large. The result is that

$$P(\text{"S"}) \rightarrow \tfrac{1}{2} \text{ as either } \begin{cases} \gamma \rightarrow \infty, \\ V \rightarrow \infty, \text{ or} \\ I \rightarrow \infty. \end{cases} \quad (29)$$

Regression toward indifference is produced as before if subjects are insensitive to time. But it is also produced by increasing the variance, or the mean and the variance, of the initial link schedule.

Just the reverse effect results from increasing the terminal link times. In Appendix E it is shown that $z'(w)$ approaches $z(w)$ as S, C get large. The maximum choice probability attainable by increasing terminal link time values, maintaining their ratio constant, is just the value appropriate to the earlier formulation in which initial link times may be disregarded. Intuitively, if terminal links are long relative to the initial link, choice appropriately reflects this preponderance in the expectancy ratio (Equation 27a).

These limits are in the right direction for the experimental results, with regression toward indifference for long initial links (Wardlaw & Davison, 1974) and sharpening of preference with long terminal links (MacEwen, 1972; Navarick & Fantino, 1976).

The implication that choice probability may not exceed that for $z(w)$ is probably not presently testable. It would require constant bias for $\beta = 1$ and $\beta \neq 1$, and the data suggest that b and β interact. However, the selection of a better reward magnitude scale than access time, coupled with the sensitivity of the concurrent-chains technique, may eventually permit predictions at this level of precision.

Summary

The theory described here treats animal performance at asymptote in learning experiments as reflecting a discrimination of improvement in expectancy of reinforcement. In simple time-correlated schedules with periodic reinforcement, the discrimination is between local or "immediate" expectancy of reinforcement and overall expectancy of reinforcement. In choice procedures the discrimination is between expectancy of reinforcement for different magnitudes and delays of reinforcement associated with each alternative. Throughout, the discrimination rule operates on the expectancy ratio, and expectancies are based on the scalar-timing process for subjective estimates of the time to reinforcement.

When time discrimination itself is the object of study, the scalar-timing process with the expectancy ratio comparator results in Weber's law.

The expectancy ratio computation for the schedules of reinforcement applications reduces, in the time discrimination case, to the familiar likelihood ratio of signal detection theory. However, the natural sensitivity index for the scalar process is the coefficient of variation for the unit timer, γ, rather than d'.

When the variables under study are temporal only, and the response alternatives symmetric, bias in choice behavior is found to be negligible. However, when amount and probability of reinforcement are varied, bias in favor of the shorter time to reinforcement emerges. This finding suggests that there may be in fact no true bias between short and long times to reinforcement, but that the reward magnitude scale should weight small magnitudes relatively more heavily than large magnitudes. The lack of bias in appetitive control of choice is to be contrasted with the very strong bias against responding found in a similar (and compatible) treatment of shock density discrimination in avoidance performance (Gibbon, 1972). Presumably, the large bias reflects the asymmetry of the response–nonresponse dimension in avoidance. It is paralleled in the present treatment of point estimation in periodic reinforcement schedules by a similar, though less extreme, bias in favor of pausing.

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Index to Appendixes

Appendix A

Activity Distribution

For a given estimate, x, of T, the rate of activity at a point, t, within the interval takes on the early, interim, or terminal rate according as t is below, within, or above the time window for interim activity, (y_I, y_F).

$$\bar{R}_T(t|x) = \begin{cases} \bar{R}_B, & t < y_I, \quad \text{(A1a)} \\ \bar{R}_I, & y_I < t < y_F, \quad \text{(A1b)} \\ \bar{R}_F, & y_F < t. \quad \text{(A1c)} \end{cases}$$

But these three rates then occur with probabilities associated with the appropriate integral of the timing distribution. Integrating out x, rate at time, t, is

$$\begin{aligned} \bar{R}_T(t) = {} & \bar{R}_B[1 - F_T(t/K_I)] \\ & + \bar{R}_I[F_T(t/K_I) - F_T(t/K_F)] \\ & + \bar{R}_F[F_T(t/K_F)], \quad \text{(A2)} \end{aligned}$$

where $K_J = 1 - 1/b_J$, $J = I, F$ (Equation 3). If the estimation process is scalar, then the distribution function values in Equation A2 may be transformed to their counterparts for the unit timer. Letting $u = t/T$,

$$\begin{aligned} \bar{R}_T(t) = {} & \bar{R}_B[1 - F(u/K_I)] \\ & + \bar{R}_I[F(u/K_I) - F(u/K_F)] \\ & + \bar{R}_F[F(u/K_F)] \\ = {} & \bar{R}(u), \quad \text{(A3)} \end{aligned}$$

where $\bar{R}(u)$ is the rate function for periodic reinforcement every (1) unit of time. Total activity is

$$R(T) \equiv \int_0^T \bar{R}_T(t)dt,$$

which, by the transformation $z = t/T$, is

$$R(T) = T \int_0^1 \bar{R}(z)dz = TR(1). \quad \text{(A4)}$$

Thus, defining an activity density function, $g_T(t)$, as the proportion of total activity at time t,

$$\begin{aligned} g_T(t) \equiv \frac{\bar{R}_T(t)}{R(T)} &= \frac{\bar{R}(u)}{TR(1)} \\ &= \frac{1}{T} g(u), \quad \text{(A5)} \end{aligned}$$

which shows that activity density is scalar in T.

Appendix B

Moments of Scalar Mixtures

As $y = Kx$, $K = 1 - 1/b$ (Equation 3), the nth raw moment of the distribution of y is

$$\mu_n'(y; T) = K^n \mu_n'(x; T), \quad \text{(B1)}$$

where $\mu_n'(x; T)$ is the nth raw moment of the mixture of estimates $f_T(x)$. The moments of the mixture in turn are obtained by integrating $x^n f_\tau(x)$ under the integral in Equation 8. With the change of variable, $u = x/\tau$ for the unit timer, this becomes simply,

$$\mu_n'(x; T) = \mu_n' \mu_n^{*\prime}(T), \quad \text{(B2)}$$

where μ_n' is the nth raw moment of the unit timer, and $\mu_n^{*\prime}(T)$ is the nth raw moment of the mixing distribution of reinforcement times, $f_T^*(\tau)$. Substituting in Equation B1, the first two moments of the distribution of y are

$$\mu(y; T) = K\mu\mu^*(T), \text{ and} \quad \text{(B3a)}$$

$$\begin{aligned} \mu_2'(y; T) &= K^2 \mu_2' \mu_2^{*\prime}(T), \text{ or} \\ \sigma^2(y; T) + \mu^2(y; T) &= K^2(\sigma^2 + \mu^2) \\ &\quad \times [\sigma^{*2}(T) + \mu^{*2}(T)], \quad \text{(B3b)} \end{aligned}$$

where μ and σ, and $\mu^*(T)$ and $\sigma^*(T)$ are the mean and standard deviation of the unit timer and the reinforcement distribution for T, respectively.

Appendix C

Weber's Law for Discrete Uniform Mixtures

Let

$$S = \{T_i\}_{i=1,2,\ldots,m}, \quad L = \{T_j\}_{j=m+1,\ldots,2m},$$

$$P(T_i) = P(T_j) = \frac{1}{2m}, \quad P(S) = P(L) = \tfrac{1}{2}.$$

The S, L, mixtures are then

$$f_S(x) = \frac{1}{m}\sum_{i=1}^{m} f_{T_i}(x)$$

$$f_L(x) = \frac{1}{m}\sum_{j=m+1}^{2m} f_{T_j}(x). \tag{C1}$$

The expectancy ratio rule then becomes

$$l_{S,L}(x) = \frac{\sum_{i=1}^{m} \frac{1}{T_i} f(x/T_i)}{\sum_{j=m+1}^{2m} \frac{1}{T_j} f(x/T_j)} > b, \tag{C2}$$

assuming $\beta = P(L)p_L^*/P(S)p_S^* = 1$. Thus for x generated by $T \in S$,

$$P(\text{"S"} \mid T) = P[l_{S,L}(x) > b]. \tag{C3}$$

Weber's law consists in showing that the short report probability (Equation C3) is unchanged when

$$S' = \{T_i'\}_{i=1,2,\ldots,m}, \quad L' = \{T_j'\}_{j=m+1,\ldots,2m},$$
$$\text{for} \quad T_k' = KT_k, \quad k = 1, 2, \ldots, 2m.$$

For x' generated by $T' = KT$, the unit timer form of the likelihood ratio, (Equation C2), is, with $u' = x'/T'$,

$$l_{S',L'}(x') = \frac{\sum_{i=1}^{m} \frac{1}{T_i'} f(u'T'/T_i')}{\sum_{j=m+1}^{2m} \frac{1}{T_j'} f(u'T'/T_j')}$$

$$= \frac{\sum_{i=1}^{m} \frac{1}{T_i} f(u'T/T_i)}{\sum_{j=m+1}^{2m} \frac{1}{T_j} f(u'T/T_j)} > b. \tag{C4}$$

But then, similarly to the two-stimulus case, (Equation 17), Equations C4 and C2 define the same region of integration on the unit timer variates, u' and $u = x/T$, respectively. Thus,

$$P(\text{"S'"} \mid T' \in S') = P(\text{"S"} \mid T \in S). \tag{C5}$$

Appendix D

Distribution of u/v

We require the probability that $u/v > w$. Fixing w, and assuming $v > 0$, this is equivalent to $t = u - wv > 0$. Since u, v, are independent samples from the unit timer, which is normal, (μ, σ), t is normal with

$$E(t) = \mu(1 - w), \tag{D1a}$$

$$Var(t) = \sigma^2(1 + w^2). \tag{D1b}$$

Letting $z = t - E(t)/\sqrt{Var(t)}$, $t > 0$ implies

$$z > z(w) = (w - 1)/\gamma\sqrt{1 + w^2}, \tag{D2}$$

where $z(w)$ is the Geary z variate (Geary, 1930). Thus,

$$P\left(\frac{u}{v} > w\right) = 1 - \Phi[z(w)], \tag{D3}$$

where Φ is the unit normal distribution function. The density of w is obtained as

$$f(w) = \frac{d}{dw}\Phi[z(w)]$$

$$= \frac{1}{\gamma\sqrt{2\pi}}\left(\frac{1 + w}{(1 + w^2)^{\frac{3}{2}}}\right) \times \exp\left[-\frac{1}{2}\left(\frac{w - 1}{\gamma\sqrt{1 + w^2}}\right)^2\right]. \tag{D4}$$

Appendix E

Approximation for Distribution of $r_{S,C}(t)$

From Equation 27b when x_I underestimates the initial link time, τ, and the elapsed time in the initial link exceeds the estimate, choice responding depends only on the terminal link estimates,

$$P(\text{"S"}) = P\left(\frac{x_C}{x_S} > b\beta\right). \tag{E1}$$

The approximation developed here regards this situation as typical of choice throughout the initial link except that variance in the estimates, x_I, degrades the precision with which the discrimination between S and C may be

made. Specifically, $r_{S,C}(t)$ is approximated by $r_{S,C}(t_I')$ where t_I' is a "typical" value of $t < x_I$, τ.[15] We may think of t_I' as the typical t value for an average of Equations 27a and 27b. It will be close to x_I, since Equation 27b requires $r_{S,C}(t) = r_{S,C}(x_I)$ when $x_I < t < \tau$.

With this construction of the expectancy ratio,

$$r_{S,C}(t) \simeq r_{S,C}(t_I') = \left(\frac{1}{\beta}\right)\left(\frac{x_C + x_I - t_I'}{x_S + x_I - t_I'}\right), \quad \text{(E2)}$$

and the Geary z variate may be obtained in a manner analogous to Appendix D. The bias criterion will be exceeded when

$$x_C - b\beta x_S + (1 - b\beta)x_I > (1 - b\beta)t_I'. \quad \text{(E3)}$$

The mean and variance of the mixture of initial link estimates, $f_I(x_I)$ may be found via Equations 9, with the scalar property as

$$E(x_I) = \mu I, \text{ and} \quad \text{(E4)}$$

$$Var(x_I) = E(\sigma^2\tau^2) + Var(\mu\tau) = \sigma^2\left[I^2 + \left(1 + \frac{1}{\gamma^2}\right)Var(\tau; I)\right], \quad \text{(E5)}$$

where μ, σ, γ are the parameters of the unit timer, and $Var(\tau; I)$ is the variance of the mixing distribution, $f_I^*(\tau)$. This distribution, imposed by the experimenter, is generally simply rescaled for different initial link mean times (Catania & Reynolds, 1968) so that $Var(\tau; I) = I^2V^2$, where $V^2 = Var(\tau/I)$. Thus, Equation E5 becomes

$$Var(x_I) = \sigma^2 I^2\left[1 + \left(1 + \frac{1}{\gamma^2}\right)V^2\right]. \quad \text{(E6)}$$

Now, x_I is demonstrably not normal in form. However, if I is not very large and if x_C, x_S are normal, the linear combination on the left of the criterion (Equation E3) is approximately normal. Letting $z = x_C - b\beta x_S + (1 - b\beta)x_I$,

$$E(z) = \mu[C - b\beta S + (1 - b\beta)I], \quad \text{(E7)}$$

and

$$Var(z) = \sigma^2[C^2 + (b\beta S)^2 + I^2(1 - b\beta)^2A(V,\gamma)], \quad \text{(E8)}$$

where $A(V,\gamma) = 1 + (1 + 1/\gamma^2)V^2$ is a constant multiplier of I^2 reflecting the contribution of experimenter-imposed (V) and subject-imposed (γ) variance in the initial link time estimates. $A(V,\gamma) \geqslant 1$, and $A(V,\gamma) \simeq 1$ for large γ and small V.

Forming the unit normal variate,

$$z' = [z - E(z)]/\sqrt{Var(z)}, \quad \text{(E9)}$$

the criterion (Equation E3) will be met when

$$z' > \frac{(1 - b\beta)(t_I' - \mu I) - \mu(C - b\beta S)}{\sigma\sqrt{C^2 + (b\beta S)^2 + I^2(1 - b\beta)^2A(V,\gamma)}}. \quad \text{(E10)}$$

We may now allow t' to approach $E(x_I) = \mu I$, and dividing numerator and denominator of (E10) by μC,

$$z' > z'(w), \quad \text{(E11)}$$

where

$$z'(w) = \frac{w - 1}{\gamma\sqrt{1 + w^2 + (I/C)^2(1 - b\beta)^2A(V,\gamma)}}, \quad \text{(E12)}$$

is the modified Geary z variate, for $w = b\beta S/C$. The new criterion, $z'(w)$, is just the original Geary z variate with an added variance term representing the contribution of variance in x_I relative to x_S and x_C. The discrimination is made difficult, as before, if subjects are insensitive to time (γ large), but also when experimenter variance, V^2, is large. That is, as long as $b\beta \neq 1$,

$$z'(w) \to 0 \text{ as } \begin{cases} \gamma \to \infty, \\ V \to \infty, \text{ or} \\ I \to \infty. \end{cases} \quad \text{(E13)}$$

Under any of these conditions, then,

$$P(\text{"S"}) \to \tfrac{1}{2}.$$

Just the reverse effect, an increase in the slope of the choice function, occurs when S, C are increased, keeping their ratio, S/C, constant. Then the added variance term vanishes, so that

$$z'(w) \to z(w) \quad \text{as} \quad S, C \to \infty, \quad S/C = \text{constant}, \quad \text{(E14)}$$

where $z(w)$ is the original Geary z variate (Equation D2). Since $|z'(w)| \leqslant |z(w)|$, the result is a sharpening of discrimination performance.

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[15] For fixed x_I, τ, the mean value theorem guarantees the existence of a t_x' for $t < x_I < \tau$ and t_τ' for $t < \tau < x_I$.